# CHE 318 Lecture 06

**Steady State Mass Transfer – Other Geometries**

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## Recap

- Diffusion in liquid and solid
  - Much slower than gas ($D_{AB}$ orders of magnitudes smaller)
  - Theories to predict $D_{AB}$ in liquid (Stokes-Einstein, Wilke-Chang)
  - Effective diffusivity in solid (void fraction ; tortuosity )
- Solving $N_A$ in liquid
  - EMCD: rare case
  - Stgnant B: approximating $x_{Bm}$
  - Calculating $c_{av}$ concentration
- Solving $N_A$ in solid (thin membrane permeability)
  - Use of solubility $S$
  - Permeability $P_M = SD_{AB}$

## Learning Outcomes

After today's lecture, you will be able to:

- **Identify** the difference between diffusion in various geometries
- **Describe** the approximations used for simplifying the soluton for $N_A$
- **Recall** the reason to choose stagnant B solution for spherical diffusion
- **Solve** pseudo-steady state diffusion problem
- **Analyze** solutions of P.S.S solution and calculate diffusivity through experiment

## Examples for Diffusion in Liquid and Solid

### Example 1: gas diffusion through liquid film

*Adapted from Geankoplis Problem 6.3-2*

Diffusion of Ammonia in an Aqueous Solution. An ammonia (A)–water (B) solution at 278 K and 4.0 mm thick is in contact at one surface with an organic liquid at this interface. The concentration of ammonia in the organic phase is held constant and is such that the equilibrium concentration of ammonia in the water at this surface is 2.0 wt % ammonia (density of aqueous solution 991.7 kg/m3) and the concentration of ammonia in water at the other end of the film 4.0 mm away is 10 wt % (density 961.7 kg/m3). Water and the organic are insoluble in each other. The diffusion coefficient of NH3 in water is $1.24 \times 10^{-9}$ $\mathrm{m}^2/\mathrm{s}$.

a. At steady state, calculate the flux $N_A$ in kg $\mathrm{mol/s} \cdot \mathrm{m}^2$
b. Calculate the flux $N_B$. Explain.

### Example 1: solutions

See handwritten notes for step-by-step solutions.

Tip

1. Determine the type of system if water-organic layer is inpenetratable (answer question b first)
2. Refere to Lecture 1 for conversion between wt%, molar fraction and concentration.

Answer:

a. $N_A = 1.52 \times 10^6$ kg $\mathrm{mol} \cdot \mathrm{m}^{-2} \cdot \mathrm{s}^{-1}$
b. $N_B = 0$ (diffusion through stagnant film)

### Example 2: diffusion through solid film (solubility)

*Adapted from Geankoplis Problem 6.5-1*

A flat plug 30 mm thick having an area of $4.0 \times 10^{-4}$ $\mathrm{m}^2$ and made of vulcanized rubber is used for closing an opening in a container. The gas CO2 at 25 °C and 2.0 atm pressure is inside the container. Calculate the total leakage or diffusion of CO2 through the plug to the outside in kg mol CO2/s at steady state. Assume that the partial pressure of CO2 outside is zero. From Barrer (B5) the solubility of the CO2 gas is 0.90 $m^3$ gas (at STP of 0°C and 1 atm) per m3 rubber per atm pressure of CO2. The diffusivity is $0.11 \times 10^{-9}$ $\mathrm{m}^2 2/\mathrm{s}$.

### Example 2: solution

> 💡 Tip
>
> 1. Notice the unit for symbols when applying $c_A = SP_A/(22.414)$
> 2. What unit should **total leakage** have?

Answer:

a. Total leakage rate $1.178 \times 10^{-13}$ kg mol $\cdot$ s$^{-1}$

### Example 3: use of permeability

*Adapted from Geankoplis Problem 6.5-3*

The gas hydrogen is diffusing through a sheet of vulcanized rubber 20 mm thick at 25 °C. The partial pressure of H2 is 1.5 atm inside and 0 outside. Using the data from Table 6.5-1 (see below), calculate the following:

a. The diffusivity $D_AB$ from the permeability $P_M$ and solubility $S$ , and compare with the value in the table.
b. The flux $N_A$ of H2 at steady state.

*Table 6.5-1. Diffusivities and Permeabilities in Solids*

| Solute (A) | Solid (B) | T (K) | $D_{AB}$, Diffusion Coefficient [m2/s] | Solubility, S $\left[\frac{m^3 \text{ solute(STP)}}{m^3 \text{ solid} \cdot atm}\right]$ | Permeability, PM $\left[\frac{m^3 \text{ solute(STP)}}{s \cdot m^2 \cdot atm/m}\right]$ | Ref. |
|---|---|---|---|---|---|---|
| $H_2$ | Vulcanized rubber | 298 | $0.85(10^{-9})$ | 0.040 | $0.342(10^{-10})$ | (B5) |

### Example 3: solution

Answer:

a. Use $D_{AB} = P_M/S$ (note the unit)
b. $N_A = 1.144 \times 10^{-10}$ kg mol $\cdot$ m$^{-2} \cdot$ s$^{-1}$

# Solving Diffusion Through Varying Area

## Diffusion Through Varying Cross-Sectional Area

### Steady state mass balance

Many industrial applications involde 1D transport with variable area $A(z)$, with mass balance:

$$[\text{In}] - [\text{Out}] = 0 \tag{1}$$
$$N_{A1}A_1 - N_{A2}A_2 = 0 \tag{2}$$

Define the **total molar flow rate** of A:

$$\overline{N}_A = N_A\, A(z)$$

At steady state: - $\overline{N}_A$ is constant - $N_A$ varies with position if $A(z)$ (or $A(r)$) varies

## Applications of Variable-Area Diffusion

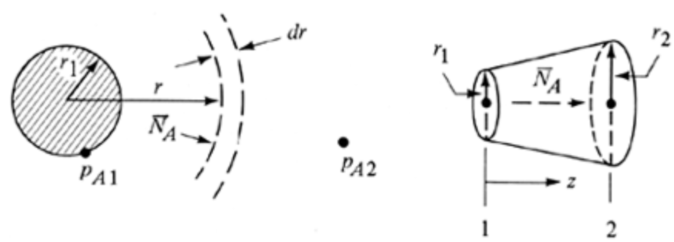


*Figure 6.2-3. Diffusion through a varying cross-sectional area: (a) from a sphere to a surrounding medium, (b) through a circular conduit that is tapered uniformally.*

This framework can be used to solve diffusion through:

- Sphere
- Cylinder
- Tube with varying diameter
- Any system with a known area function $A = A(z)$ or $A = A(r)$

### Key ideas

- Do **not** solve as simple 1D Cartesian diffusion
- Geometry enters through $A(z)$
- Flux adjusts to maintain constant $\overline{N}_A$

## Summary

- Solving various examples in liquid and gas diffusion systems
- Use of weight %, log-mean molar ratio, solubility and permeability