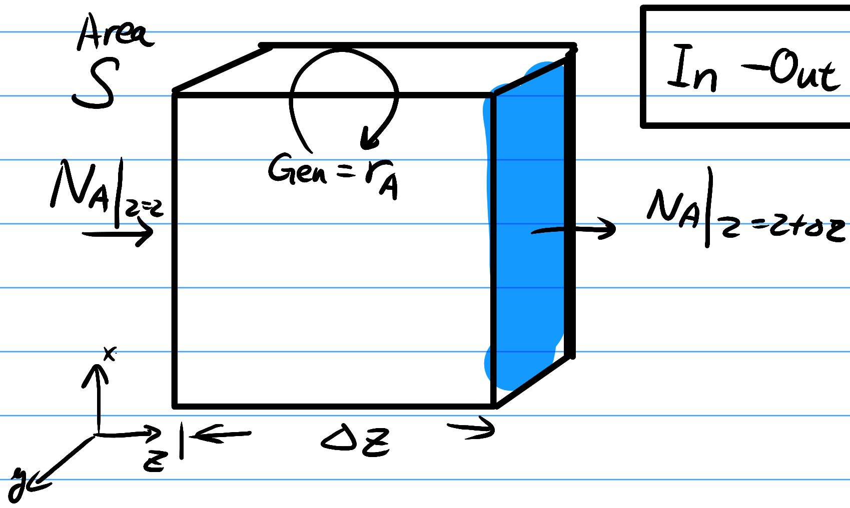
CHE 318 Lecture 09

Jan-21-2026

U.S.S mass transport



$$\boxed{\text{In} - \text{Out} + \text{Gen} = \text{Acc}}$$

Derivation of govern eqn

$$N_A|_{z=z} - N_A|_{z=z+\Delta z} = -\left(\frac{N_A|_{z+\Delta z} - N_A|_{z}}{\Delta z}\right) \cdot \Delta z$$

$\Downarrow$ Def. of derivative

$$= -\frac{\partial N_A}{\partial z} \cdot \Delta z$$

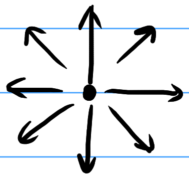
$$\Rightarrow \quad \frac{\partial c_A}{\partial t} = -\nabla \cdot N_A + r_A$$

1D $\quad \dfrac{\partial c_A}{\partial t} = -\dfrac{\partial N_A}{\partial z} + r_A$

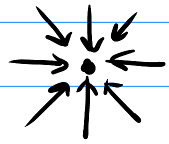
Divergence operator ∇· vector ⇒ scalar

$$\nabla\cdot\vec{N}_A = \frac{\partial \vec{N}_A}{\partial x} + \frac{\partial \vec{N}_A}{\partial y} + \frac{\partial \vec{N}_A}{\partial z}$$

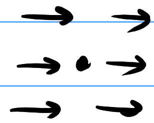
$\nabla\cdot\vec{N}_A > 0$



$\nabla\cdot\vec{N}_A < 0$



$\nabla\cdot\vec{N}_A = 0$



(see similarity with 1D S.S)

Gradient ∇ scalar → gradient

$$\nabla C \rightarrow \left(\frac{\partial c}{\partial x}, \frac{\partial c}{\partial y}, \frac{\partial c}{\partial z}\right)$$

Laplace operator ∇^2scalar → scalar

$$\nabla^2 c \longrightarrow \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2}$$

Special cases for U.S.S

$$\boxed{\frac{\partial c}{\partial t} = r_A - \nabla\cdot(-D_{AB}\nabla C_A + C_A v_m)}$$

Only 1 governing eq, different simplification

① Fick's second law

$$\begin{cases} v_m = 0 \Rightarrow \text{EMCD flux} \\ D_{AB} \text{ constant} \\ r_A = 0 \end{cases}$$

$$\frac{\partial c}{\partial t} = 0 - \nabla\cdot(-D_{AB}\nabla C_A + 0)$$

no negative sign

$$= D_{AB}\nabla^2 C_A \Longrightarrow \text{Fick's } 2^{nd} \text{ law}$$

Fick's 1^{st} law $J^*_{Az} = - D_{AB}\frac{\partial C_A}{\partial z}$

↑negative sign

② Constant D_{AB} (no other simplification)

$$\frac{\partial c_A}{\partial t} = D_{AB}\nabla^2 c_A - \nabla\cdot(c_A \vec{v}_m) + r_A$$

$$= D_{AB}\nabla^2 c_A - c_A \nabla\cdot\vec{v}_m - \nabla c_A\cdot\vec{v}_m + r_A$$

$$\nabla\cdot(\lambda\vec{u}) = \boxed{\nabla\lambda}\cdot\boxed{\vec{u}} + \boxed{\lambda}\,\boxed{\nabla\cdot\vec{u}}$$

→ vector

scalar

③ case ② But incompressible ($\nabla\cdot\vec{v}_m = 0$)

$$\frac{\partial c_A}{\partial t} = D_{AB}\nabla^2 c_A - \nabla c_A\cdot\vec{v}_m + r_A$$