CHE 318 L11

Jan -28-2026

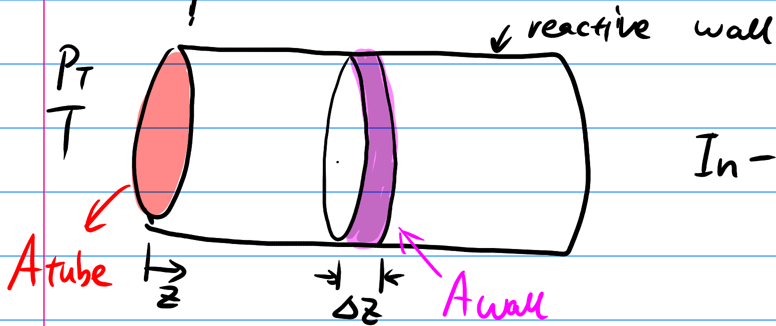
Recap: Unsteady state mass transfer

$$\frac{\partial C_A}{\partial t} = r_A - \frac{\partial N_A}{\partial z} \quad (1D)$$

Last lecture we solved evaporation into stagnant B (Arnold problem)

We will see examples of reaction

Example 1 Reactive wall in a conduit



In − Out + Gen = Acc

I. Note difference in arears

$$A_{tube} N_A\big|_z - A_{tube} N_A\big|_{z+\Delta z} + A_{wall} \cdot r_A = A_{tube} \cdot \Delta z \cdot \frac{\partial C_A}{\partial t}$$

$$\left.\begin{aligned} A_{tube} &= \frac{\pi D^2}{4} \\ A_{wall} &= \Delta z \cdot \pi D \end{aligned}\right\} \Rightarrow \text{M.B. becomes}$$

$$\frac{\cancel{\pi D^2}}{\cancel{4}} \cdot \left(-\frac{\partial N_A}{\partial z}\right) + \frac{4\cancel{\pi D} \cdot r_A}{D^2} = \frac{\partial C_A}{\partial t} \cdot \frac{\cancel{\pi D^2}}{\cancel{4}}$$

$$-\frac{\partial N_A}{\partial z} + \frac{4}{D} r_A = \frac{\partial C_A}{\partial t}$$

$$-\frac{\partial N_A}{\partial z} + \frac{4k}{D}(C_{As} - C_A) = \frac{\partial C_A}{\partial t}$$

II. Link with flux eq

$$N_A = -D_{AB}\frac{\partial C_A}{\partial z} + C_A \cdot v_m$$

Const v_m

$$\partial N_A/\partial z = -D_{AB}\frac{\partial^2 C_A}{\partial z^2} + \cancel{C_A \frac{\partial v_m}{\partial z}} + \frac{\partial C_A}{\partial z}\cdot v_m$$

III Combine I & II

$$\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial z^2} - v_m \cdot \frac{\partial C_A}{\partial z} + \frac{4k}{D}(C_{As} - C_A)$$

B.C.

$z=0 \quad C_A = C_{A0} \qquad z=L \quad C_A = C_A(z)$ (to be determined)

Practically, question is: given k, v_m, D, how to determine Length of tube so that $C(z=L)$ lower than threshold?

To solve we need simlifications

1. $v_m \gg v_{Ad}$ convection $\gg$ diffusion
2. steady state (long-term CO_2 removal)

$$v_m \frac{dC_A}{dz} = -\frac{4k}{D}(C_{As} - C_A) \qquad \text{Solve ODE}$$

$$-\frac{dC_A}{C_{As} - C_A} = \frac{4k}{v_m D} dz$$

diameter, not D_{AB}

Integrate $$\int_{C_{A0}}^{C_A(z)} -\frac{dC_A}{C_{As} - C_A} = \int_0^z \frac{4k}{v_m D} dz'$$

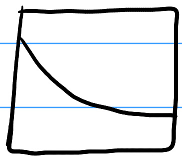
$$\Rightarrow \quad C_A(z) = C_{A,s} - (C_{A,s} - C_{A0})\exp\left(-\frac{4k}{v_m D}\cdot z\right)$$

Insights: With reaction

1) At the end of pipe $\longrightarrow C_0 \geq C_{A,S}$

$C_{A,S}$ must be low to allow efficient absorption

2) Concentration decays exponentially



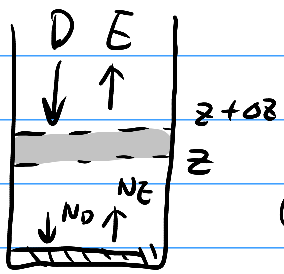
decay length $\approx \dfrac{D\, v_m}{4k}$ $\quad \dfrac{m \cdot m/s}{4 \cdot k} \Rightarrow m$

What is unit of k? k in unit $[m]/[s]$

3) $N_A \doteq C_A \cdot v_m \neq$ Constant !

Example 3. Catalyst conversion $D \rightarrow E$ in a well

Instantaneous conversion $\rightarrow x_{D0} = 0$



$$m\, D \longrightarrow n E$$

① M.B in control vol

$$In - Out + \cancel{Gen} = Acc$$

$$-\frac{\partial N_D}{\partial z} = \frac{\partial C_D}{\partial t}$$

We don't have [gen] in control volume (reaction only at bottom)

② Flux equ

$$N_D = -D_{DE}\frac{\partial C_D}{\partial z} + \frac{C_D}{C_T}(N_D + N_E)$$

③ Relation between N_D & N_E

$\downarrow N_D \quad \uparrow N_E$ Instantaneous reaction

Instantaneous = Reaction rate controlled by diffusion

Complete = Conversion between D & E follow stoichiometry

$$mD \longrightarrow nE \qquad \frac{N_D}{m} + \frac{N_E}{n} = 0$$

$$N_E = -\frac{n}{m} N_D$$