CHE318 L15
Feb-04 2026

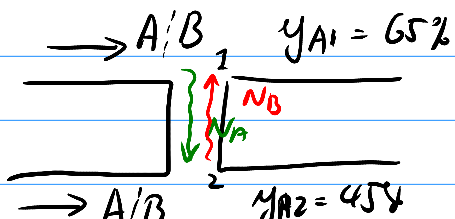
Recap: Boundary condition

$$K = \frac{C_{\text{interface } 1}}{C_{\text{interface } 2}}$$

k_c': M.T. coefficient $N_A = k_c'(C_{Li} - C_i)$

Various k_c forms: k_c k_c' k_x k_x' k_y k_y'

Solution to Q1



1 → 2

$$N_A = k_y' \cdot (y_{A1} - y_{A2})$$
$$= 1.5\times10^{-4} \times (0.65 - 0.45)$$
$$= 3.0\times10^{-5}\ \text{kg mol}/(\text{m}^2\cdot\text{s})$$

$N_B = -N_A$ 2 → 1

We realize k_y' is dependent on total pressure!

$k_y' \rightarrow k_c'$ $k_y' = k_c' \cdot C_T = k_G' \cdot P_T$

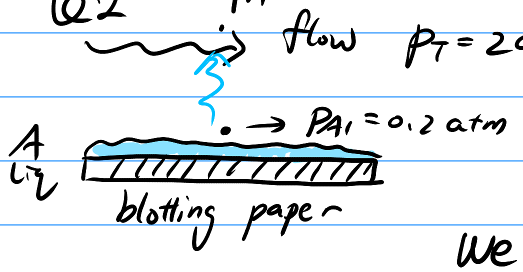
$$k_G' = \frac{k_y'}{P_T} = \frac{1.5\times10^{-4}}{700\ \text{kPa}} = 2.14\times10^{-7}\ \text{kg mol}/(\text{m}^2\cdot\text{s}\cdot\text{kPa})$$

$$k_c' = \frac{k_y'}{C_T} = \frac{k_y'}{P_T}\cdot RT = k_G'\cdot RT = 6.65\times10^{-4}\ \text{m/s}$$

superficial vel

Q2

$p_{A2} = 0$ $\quad$ $P_T = 2\,atm$



$k_y' = 6.78 \times 10^{-5} \dfrac{kg\,mol}{m^2 \cdot s}$

We don't care direction of N_A in fluid

All mass transfer coefficient $\Rightarrow$ lumped

$$N_A = k_y' \cdot \frac{1}{y_{Bm}} \cdot (y_{A1} - y_{A2})$$

stagnant B

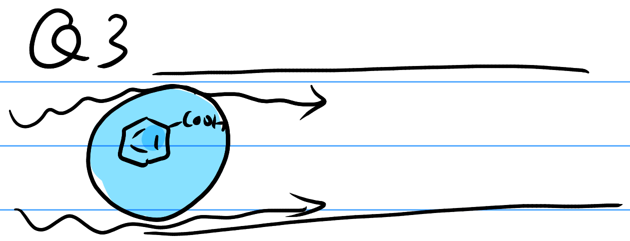
$y_{Bm} \approx \dfrac{y_{B1} + y_{B2}}{2} = 0.95$ $\quad y_{A1} = 0.1 \quad y_{A2} = 0$

$N_A = 6.78 \times 10^{-5} \cdot \dfrac{1}{0.95} \cdot 0.1 = 7.14 \times 10^{-6}\ kg\,mol/(m^2 \cdot s$

$k_y = k_y' / y_{Bm} = 7.14 \times 10^{-5}\ kg\,mol/(m^2 \cdot s)$

$k_G = k_y / P_T = 3.57 \times 10^{-5}\ kg\,mol/(m^2 \cdot s \cdot atm)$

Q3



$$\overline{Flux} = \frac{[\text{weight}]}{[\text{molar mass}][\text{time}]}$$

$$N_A \cdot 4\pi r_o = \frac{m_1 - m_2}{M_A \cdot \Delta t} = k_L (C_{As} - 0)$$

$$k_L = \frac{1}{4\pi r_o^2} \cdot \frac{m_1 - m_2}{C_{As} \cdot M_A \cdot \Delta t}$$

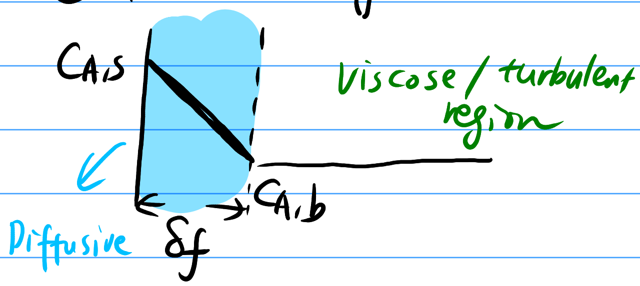
← Get from solubility

CHE318 L15

Theories of Mass Transfer Coefficient

Different theories of k_c'

① Film theory



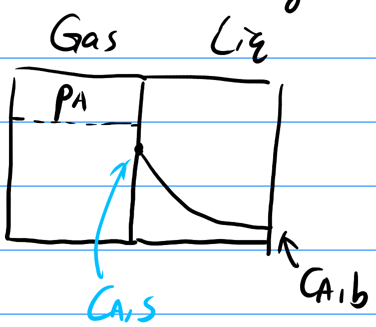
$$N_A = J^*_{Az} = \underbrace{\boxed{\frac{D_{AB}}{\delta_f}}}_{k_c'}(C_{A,s} - C_{A,b})$$

1) Physically intuitive ✓

2) $k_c' \propto D_{AB}^{1.0}$ ✗ (Chilton - Colburn result $k_c' \propto D_{AB}^{\frac{2}{3}}$)

② Penetration theory

Dankwerts 1950



Solve $D_{AB}\dfrac{\partial^2 C_A}{\partial z^2} = \dfrac{\partial C_A}{\partial t}$

B.C. $\begin{cases} C_A(z=0) = C_{A,s} \\ C_A(z=\infty) = C_{A,b} \end{cases}$

Solution $$\frac{C_A(z,t) - C_{A,b}}{C_{A,s} - C_{A,b}} = 1 - \mathrm{erf}\left(\frac{z}{\sqrt{4D_{AB}t}}\right)$$

$$\mathrm{erf} = \frac{2}{\sqrt{\pi}}\int_0^z e^{-x^2}dx$$

$$N_A = -D_{AB}\left.\frac{\partial C_A}{\partial z}\right|_s$$

$$= \sqrt{\frac{D_{AB}}{\pi t}}(C_{A,s} - C_{A,0})$$

↑ depend on time

$$\frac{\partial C_A}{\partial z} = -\frac{2}{\sqrt{\pi}}\cdot(C_{A,s} - C_{A,b})\cdot\frac{1}{\sqrt{4D_{AB}t}}\cdot e^{-\left(\frac{z}{\sqrt{4D_{AB}t}}\right)^2}$$

$\int t^{-\frac{1}{2}} = 2t^{\frac{1}{2}} + \text{Const}$

If contact time is short

$$\Rightarrow \underset{\text{(average)}}{N_A} = \frac{\int_0^{t_U} N_A|_{z=0}\, dt}{t_L} = \underbrace{\boxed{\sqrt{\frac{4D_{AB}}{\pi t_U}}}}_{k_c'} (C_{A,s} - C_{A,b})$$

t_U = penetration time scale

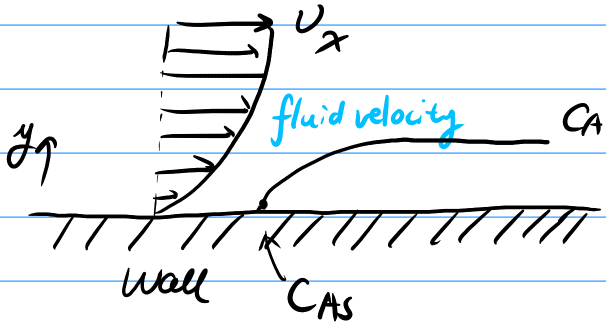
Penetration theory $\Rightarrow k_c' \propto \sqrt{D_{AB}}$ (close to practical $n = 0.8 \sim 0.9$)

Danckwerts $\Rightarrow$ { modified factor for turbulent m.t. / surface renewal

$$k_c' = \sqrt{D_{AB} \cdot s}$$

s ↑ renewal factor

③ Boundary layer theory



① Developed for laminar flow

② Intuition:

1) N_A has x, y component

2) $\rightarrow x$ convection only

$\uparrow y$ diffusion / conv

3) velocity (v_x, v_y) known

Mass balance

$$In_x + In_y - Out_x - Out_y = Acc$$

$$\underbrace{v_x \frac{\partial C_A}{\partial x} + v_y \frac{\partial C_A}{\partial y}}_{\substack{\vec{U} \cdot \nabla C_A \\ \text{Convection div}}} = \underbrace{D_{AB} \frac{\partial^2 C_A}{\partial y^2}}_{\text{Diffusive div}}$$

Boundary condition
$$\begin{cases} \left.\dfrac{v_x}{v_\infty}\right|_{y=0} = 0 \\ \left.\dfrac{C_A - C_{AS}}{C_{A,\infty} - C_{AS}}\right|_{y=0} = 0 \end{cases} \qquad \begin{cases} \left.\dfrac{v_x}{v_\infty}\right|_{y=\infty} = 1 \\ \left.\dfrac{C_A - C_{AS}}{C_{A,\infty} - C_{AS}}\right|_{y=\infty} = 1 \end{cases}$$

Laminar flow, v_x profile is

$$N_{Re,x} = \frac{x \cdot v_\infty \cdot \rho}{\mu}$$

$$\left(\frac{\partial v_x}{\partial y}\right)_{y=0} = 0.332 \frac{v_\infty}{x}\sqrt{N_{Re,x}}$$

Reynolds Number

Characteristic length, use x

$N_{sc}=1$

Same boundary

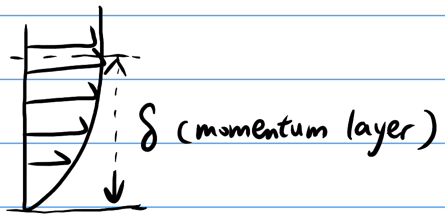
$$\left(\frac{\partial C_A}{\partial y}\right)_{y=0} = \frac{0.332}{x}\sqrt{N_{Re,x}} \cdot (C_{A\infty} - C_{As})$$

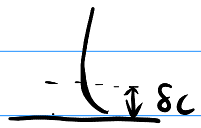
$$N_A = -D_{AB}\left(\frac{\partial C_A}{\partial y}\right)_{y=0} \Rightarrow k_c' = \frac{0.332\, D_{AB}\sqrt{N_{Re,x}}}{x}$$

Steady State

this is when $N_{sc} = 1$

$N_{sc} = \frac{x \cdot k_c'}{D_{AB}}$ Schmidt number





We have in general Boundary thickness relation

$$\boxed{\frac{\delta}{\delta_c} = N_{sc}^{\frac{1}{3}}}$$

General equ for boundary layer theory

$$\frac{k_c' x}{D_{AB}} = 0.332\, N_{Re,x}^{1/2}\, N_{sc}^{1/3}$$

$N_{sc} \propto \frac{1}{D_{AB}} \Rightarrow k_c' \propto D_{AB}^{\frac{2}{3}}$ (very close to practical results)