CHE 318 Lecture 16

Dimensionless Numbers In Mass Transfer

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Learning Outcomes

After today's lecture, you will be able to:

- Recall the nature behind dimensionless numbers
- Describe how to correlate mass transfer behaviour with dimensionless numbers
- Understand the usage of dimensionless numbers in different geometries

Recap: Boundary Layer Theory

The global mass transfer coefficient in a tube:

$$k'_c = \frac{0.664 D_{AB}}{L} N_{Re}^{0.5} N_{Sc}^{1/3} \tag{1}$$

- N_{Re}: Reynolds number
- N_{Sc}: Schmidt number

Why Do We Need These Dimensionless Numbers?

- Expressing fluxes using k coefficients are easy
- But do we need to measure k for each system specifically?
  - Of course **NO**!
- We can correlate the values of k measured in different geometries, velocities using **dimensionless numbers**
- Similar treatment exists in heat and momentum (fluid) transfer

Dimensionless Numbers In Mass Transfer

- **General form**: $N_{\text{name}} = \frac{\text{Scale of effect 1}}{\text{Scale of Effect 2}}$
- **Schmidt number** (ratio between momentum diffusivity and molecular diffusivity)

$$N_{\text{Sc}} = \frac{\mu}{\rho D_{AB}} \tag{2}$$

- **Sherwood number** (ratio between convective mass transfer and molecular mass transfer)

$$N_{\text{Sh}} = \frac{k'_c L}{D_{AB}} \tag{3}$$

- **Reynolds number** (ratio between kinetic vs viscous forces of fluid flow)

$$N_{\text{Re}} = \frac{Lv\rho}{\mu} \tag{4}$$

- L: characteristic length of system
- Location specific $N_{\text{Re},x}$ also used

How Are k'_c Correlated By Dimensionless Numbers?

- Idea: mass transfer in flowing fluid described by v, ρ, μ, c, D_{AB} and geometry (characteristic length L)
- The combinations of these properties –> dimensionless number groups (N_{Sc}, N_{Sh}, N_{Re})
- The Chilton-Colburn j-factor analog has most successful use in mass transfer

$$\begin{aligned}
j_D &= f/2 &(5)\\
&= \frac{k'_c}{v_{av}}(N_{\text{Sc}})^{2/3} &(6)\\
&= \boxed{\frac{k'_c L}{D_{AB}}}\boxed{\frac{\rho D_{AB}}{\mu}}\boxed{\frac{\mu}{L v_{av}\rho}}(N_{\text{sc}})^{2/3} &(7)\\
&= N_{\text{Sh}} N_{\text{Sc}}^{-1} N_{\text{Re}}^{-1} (N_{\text{sc}})^{2/3} &(8)\\
&= \frac{N_{\text{Sh}}}{N_{\text{Re}} N_{\text{Sc}}^{1/3}} &(9)
\end{aligned}$$

- f is the Fanning Friction Factor (can be found in table)

General Procedure To Calculate k_c'

1. Calculate Reynolds number N_{Re} from fluid properties + geometry
2. Determine flow regime (liquid)

- $N_{Re} < 2100 \rightarrow$ laminar flow
- $N_{Re} \geq 2100 \rightarrow$ turbulent flow

3. Evaluate friction factor f

- Laminar flow:

$$f = \frac{16}{N_{Re}}$$

- Turbulent flow:

$$f = \frac{\tau_s}{\frac{1}{2}\rho v^2}, \qquad \tau_s = \frac{\Delta P_f \, \pi R^2}{2\pi R \, \Delta L}$$

4. Compute mass-transfer j-factor

$$j_D = \frac{f}{2}$$

5. Obtain mass-transfer coefficient

$$k_c' = j_D \, v_{av} \, N_{Sc}^{-2/3}$$

Use of Empirical Mass Transfer Laws

- In many systems, flux and / or concentration profiles become hard to have simple form
- Luckily we can simplify typical mass transfer problems as different geometries
  - Cyliner / Pipe
  - Parallel plates
  - Flow around sphere
  - Packed bed
- We will show a few case studies for different geometries
- Dimensionless numbers (N_{Re}, N_{Sc}, N_{Sh}) help determine governing equations

Case 1: Mass Transfer for Flow Inside Pipes

- Usually use the Linton & Sherwood chart
- Valid for gas / liquid in both laminar & turbulent regimes

Flow Inside Pipes: Solution Procedure

- Governing dimensionless quantity:

$$\frac{W}{D_{AB}\rho L} = N_{Re} N_{Sc} \frac{D}{L}\frac{\pi}{4} \tag{10}$$

$$= \frac{[\text{Total Forced Flow}](\text{kg/s})}{[\text{Total Diffusive Flow}](\text{kg/s})} \tag{11}$$

- If gas use the "rodlike flow" line
- If liquid, distinguish 2 cases
  - parabolic flow ($N_{Re} < 2100;\ \frac{W}{D_{AB}\rho L} > 400$)
  - turbulent flow ($N_{Re} > 2100;\ 0.6 < N_{Sc} < 3000$)

Flow Inside Pipes: Solution For Liquid

Parabolic flow

$$\frac{c_A - c_{A,s}}{c_{A,i} - c_{A,s}} = 5.5 \left[\frac{W}{D_{AB}\,\rho\, L}\right]^{-\frac{2}{3}} \tag{12}$$

- c_A: exit concentration
- $c_{A,i}, c_{A,s}$: inlet & surface concentration
- W: flow rate in (kg/s)
- k'_c can be calculated by j_D

Turbulent flow

$$N_{Sh} = k_c' \left(\frac{D}{D_{AB}}\right) \tag{13}$$
$$= \frac{k_c\, p_{BM}}{P} \left(\frac{D}{D_{AB}}\right) \tag{14}$$
$$= 0.023 \left(\frac{\rho D v}{\mu}\right)^{0.83} \left(\frac{\mu}{\rho D_{AB}}\right)^{0.33} \tag{15}$$
$$= 0.023\, N_{Re}^{0.83}\, N_{Sc}^{0.33} \tag{16}$$

- Similar to the j_D analog
- Just need N_{Re} and N_{Sc} to determine k_c'
- Characteristic length D is pipe diameter!

Case 2: Flow Past Parallel Plates

- Can be used for gases or evaporation of liquid
- Distinguished between laminar & turbulent flow
- N_{Re} regime cutoff different in gas & liquid!
-     – Characteristic length L: length of plate in flow direction

Flow Past Parallel Plates: Results

Laminar flow ($N_{Re} < 15,000$)

$$j_D = 0.664 N_{Re,L}^{-0.5} \tag{17}$$
$$\frac{k_c' L}{D_{AB}} = 0.664 N_{Re,L}^{0.5} N_{Sc}^{1/3} \tag{18}$$

- This follows our derivation of boundary layer theory

Turbulent flow

- Gas: $15,000 < N_{Re}< 300,000 $

$$j_D = 0.036 N_{Re,L}^{-0.2} \tag{19}$$

- Liquid: $600 < N_{Re} < 50,000$

$$j_D = 0.99 N_{Re,L}^{-0.5} \tag{20}$$

Case 3: Flow Past Single Sphere

- Frequent geometry in particle solutions
- Low Reynolds regime   solution for stagnant diffusion on spherical surface
- High Reynolds regime   correct N_{Sh} and back calculate k_c'

Flow Past Single Sphere: Results

Low Reynolds ($N_{Re} < 2$)

$$N_A = \boxed{\frac{2D_{AB}}{D_p}}(c_{A1} - c_{A2}) \tag{21}$$

$$= k_c(c_{A1} - c_{A2}) \tag{22}$$

$$= \frac{k_c'}{x_{Bm}}(c_{A1} - c_{A2}) \tag{23}$$

$$\tag{24}$$

- For $x_{Bm} \approx 1$, we have:

$$k_c' = \frac{2D_{AB}}{D_p}$$

- Sherwood number: $N_{Sh} = 2$

High Reynolds ($N_{Re} > 2$)

- Gas:

$$N_{Sh} = 2 + 0.552 N_{Re}^{0.53} N_{Sc}^{1/3} \tag{25}$$

$$0.6 < N_{Sc} < 2.7 \quad N_{Re} < 48000 \tag{26}$$

- Liquid:

$$N_{Sh} = 2 + 0.95 N_{Re}^{0.5} N_{Sc}^{1/3}; \; N_{Re} < 2000 \tag{27}$$

$$N_{Sh} = 0.347 N_{Re}^{0.62} N_{Sc}^{1/3}; \; 2000 < N_{Re} < 17000 \tag{28}$$

- Back calculate $k_c' = N_{Sh} \frac{D_{AB}}{D_p}$

Case 4: Mass Transfer for Packed Beds

- Very common geometry for chemical engineering
  - Adsorption and desorption through solid particles (gases and liquids)
  - Catalytic processes with very large surface area
- Geometry characteristics: void fraction ε:

$$\varepsilon = \frac{\text{void space}}{\text{total space}} = \frac{\text{void space}}{\text{void space} + \text{solid space}}$$

  - Typically $0.3 < \varepsilon < 0.5$
  - Void fraction is difficult to measure experimentally

Correlation Equations In Packed Bed

Correlation 1, applicable to:

- gase with $10 < N_{Re} < 10,000$
- liquid with $10 < N_{Re} < 1500$

$$j_D = j_H = \frac{0.4548}{\varepsilon} N_{Re}^{(} - 0.4069) \tag{29}$$

$$N_{Re} = \frac{D_p v' \rho}{\mu} \tag{30}$$

- D_p: (average) particle diameter
- v': superficial velocity in the tube without packing

Correlation Equations In Packed Bed (II)

Correlation 2, applicable to:

- liquid with $0.0016 < N_{Re} < 55$, $165 < N_{Sc} < 70000$

$$j_D = \frac{1.09}{\varepsilon} N_{Re}^{(} - 2/3) \tag{31}$$

- liquid with $55 < N_{Re} < 1500$, $165 < N_{Sc} < 10690$

$$j_D = \frac{0.250}{\varepsilon} N_{Re}^{(} - 0.31) \tag{32}$$

Correlation Equations In Packed Bed (III)

Correlation 3, applicable to fluidized beds

- $10 < N_{Re} < 4000$ (gas & liquid)

$$j_D = \frac{0.4548}{\varepsilon} N_{Re}^(- 0.4069) \tag{33}$$

- $1 < N_{Re} < 10$ (liquid only)

$$j_D = \frac{1.1068}{\varepsilon} N_{Re}^(- 0.72) \tag{34}$$

Packed Bed Calculation Steps

1. Known value from operational column: ε, V_b (total volume), D_p, D_{AB}, μ, ρ, etc.
2. Depend on the operational range, calculate N_{Re}, N_Sc choose the equation for j_D
3. Obtain k_c from j_D value
4. Calculate flux N_A
5. Estimate effective area A_{eff} inside the columne $\overline{N}_A = A_{eff} N_A$

Caveats In Packed Bed Problems (1)

- Estimate the effective area?
  - First calculate the effective surface area per volume a then A_{eff}

$$a = \frac{6(1-\varepsilon)}{D_p} \tag{35}$$

$$A_{eff} = a V_b \tag{36}$$

Caveats In Packed Bed Problems (2)

- Use log-mean driving force correction

$$\overline{N}_A = A_{eff} N_A \tag{37}$$

$$= A_{eff} k_c \frac{(c_{A,i} - c_{A1}) - (c_{A,i} - c_{A2})}{\ln \frac{(c_{A,i} - c_{A1})}{(c_{A,i} - c_{A2})}} \tag{38}$$

where

- $c_{A,i}$: surface concentration
- c_{A1}, c_{A2}: in- and outlet concentrations

Caveats In Packed Bed Problems (3)

- Mass-flow balance

$$\overline{N}_A = A_{eff} N_A \tag{39}$$

$$= V(c_{A2} - c_{A1}) \tag{40}$$

where V is the volumetric flow rate.

These equations will give rise to solving the flow in packed bed problem.

Summary

- Dimensionless numbers can be used to correlate mass transfer problems in different flow rate, dimension etc
- Typically, start with a known geometry (pipe? parallel plate? sphere? packed bed?)
- Find the correlation with dimensionless numbers N_{Re}, N_{Sc}
- Calculate the final mass transfer rate