CHE 318 L16
Feb-11 2026

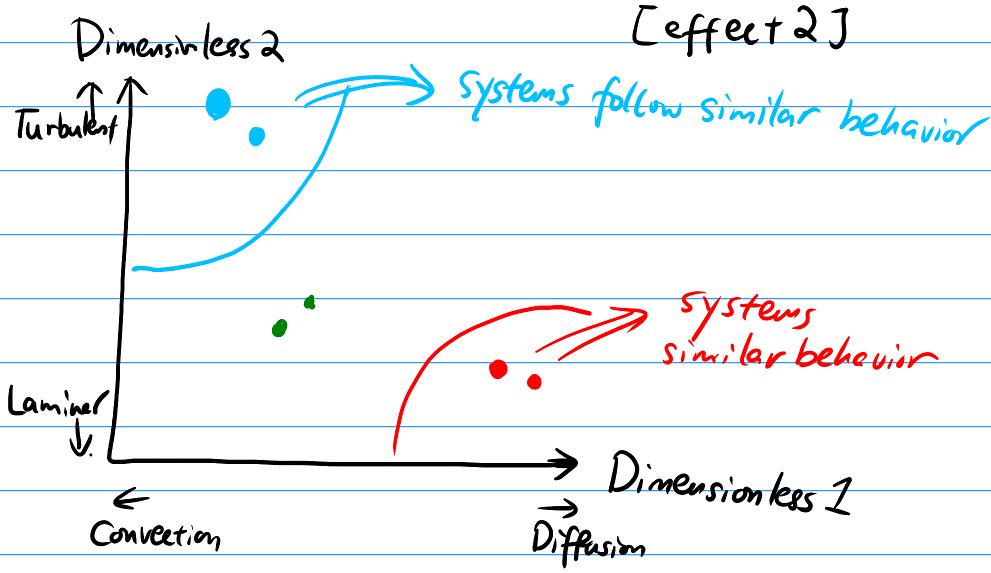
We have seen

1) k_c' ⇒ simple form to $N_A = k_c'(C_{A1} - C_{A2})$
2) k_c' can be taken from theories
   - Film
   - Penetration
   - Boundary layer

What if
1) We're interested in getting accurate k_c'
2) Measured k_c', can we applied to other geometry?

We can use "Dimensionless Analysis"

$$\text{Dimesionless number} = \frac{[\text{effect 1}]}{[\text{effect 2}]}$$



N_{Sc} (Schmidt Nr) $= \dfrac{\mu}{\rho D_{AB}} = \dfrac{[\text{momentum diff}]}{[\text{molar diff}]}$

also $\dfrac{\delta}{\delta_c} = N_{Sc}^{\frac{1}{3}}$

N_{Sh} (Sherwood Nr) $= \dfrac{k_c' L}{D_{AB}} = \dfrac{[\text{conv m.t.}]}{[\text{diff m.t.}]}$

N_{Re} (Reynolds Nr) $= \dfrac{v \rho L}{\mu} = \dfrac{vL}{\left(\frac{\mu}{\rho}\right)} = \dfrac{[\text{kinetic transp}]}{[\text{viscosity transp}]}$

How do we correlate k_c'?

Chilton-Colburn j-factor
(same factor for momentum / heat / mass transfer)

For mass transfer

$$j_D = \frac{k_c'}{v_{av}} (N_{Sc})^{\frac{2}{3}} = \frac{f}{2}$$

$f \Rightarrow$ Fanning friction factor

v_{av}: ↑ average fl. velocity

Solution process

Calculate N_{Re} (fluid alone from geometry)

$N_{Re} < 2100 \rightarrow$ Laminar flow: $f = \dfrac{16}{N_{Re}} \xrightarrow{j_D = \frac{f}{2}}$ (↑ N_{Sc}) $\boxed{\text{Get } k_c' = j_D \cdot v_{av} \left(N_{Sc}^{-\frac{2}{3}}\right)}$

$N_{Re} > 2100 \rightarrow$ Turbulent flow: $f = \dfrac{\tau_s}{\frac{1}{2}\rho v^2} = \dfrac{\Delta P_f \pi R^2}{2\pi R \Delta L} \Big/ \left(\dfrac{\rho v^2}{2}\right) \xrightarrow{j_D = \frac{f}{2}}$ Get k_c'

More generally

$$\boxed{j_D = \frac{f}{2} = \frac{N_{Sh}}{N_{Re} N_{Sc}^{\frac{1}{3}}}}$$

$N_{Sh} \rightarrow$ has k_c'

Most likely, we use dimensionless analysis on an established chart

Pipe

$\frac{W}{D_{AB}\rho L}$ determines exit concentration

$$\frac{kg/s}{\frac{m^2}{s}\cdot\frac{kg}{m^3}\cdot m}$$

$$\frac{W}{D_{AB}\rho L} = N_{Re}N_{Sc}\cdot\frac{D}{L}\cdot\frac{\pi}{4}$$

1) Gas (use rod-like flow curve)

2) Liquid $\frac{W}{D_{AB}\rho L} > 400$ parabolic flow

$$\boxed{\frac{C_A - C_{A,0}}{C_{A,i} - C_{A,0}} = 5.5\left(\frac{W}{D_{AB}\rho L}\right)^{-2/3}}$$

3) Liquid/Gas turbulent flow

$N_{Re} > 2100$ If $0.6 < N_{Sc} < 3000$

$$N_{Sh} = k_c'\left(\frac{\overset{\text{diameter}}{D}}{D_{AB}}\right) = 0.023\underbrace{\left(\frac{\rho D v}{\mu}\right)^{0.83}}_{N_{Re,D}}\underbrace{\left(\frac{\mu}{\rho D_{AB}}\right)^{0.33}}_{N_{Sc}}$$

1) (very close to the j-D analog)

2) We just need $N_{Re,D}$ N_{Sc} $\Rightarrow k_c'$

3) Only applicable for pipe! (& N_{Sc} range)

Solution procedure

Geometry ? $\xrightarrow{\text{not pipe}}$ choose other eqn.

$\downarrow$ pipe

Calculate N_{Re} $\xrightarrow{\text{laminar}}$ $\frac{W}{D_{AB}\rho L} > 400$? $\xrightarrow{Y}$ $\frac{c_A - c_{A0}}{c_{Ai} - c_{A0}} = \left(\frac{W}{D_{AB}\rho L}\right)^{-\frac{2}{3}} \cdot 5.5$

k_c' from j_D

N

$\downarrow$ turbulent

$0.6 < N_{Sc} < 3000$?

N

$\downarrow$ Y

$$k_c'\left(\frac{D}{D_{AB}}\right) = 0.023\, N_{Re}^{0.83}\, N_{Sc}^{0.33}$$

Case II Fl