CHE 318 L17

Dimension less numbers

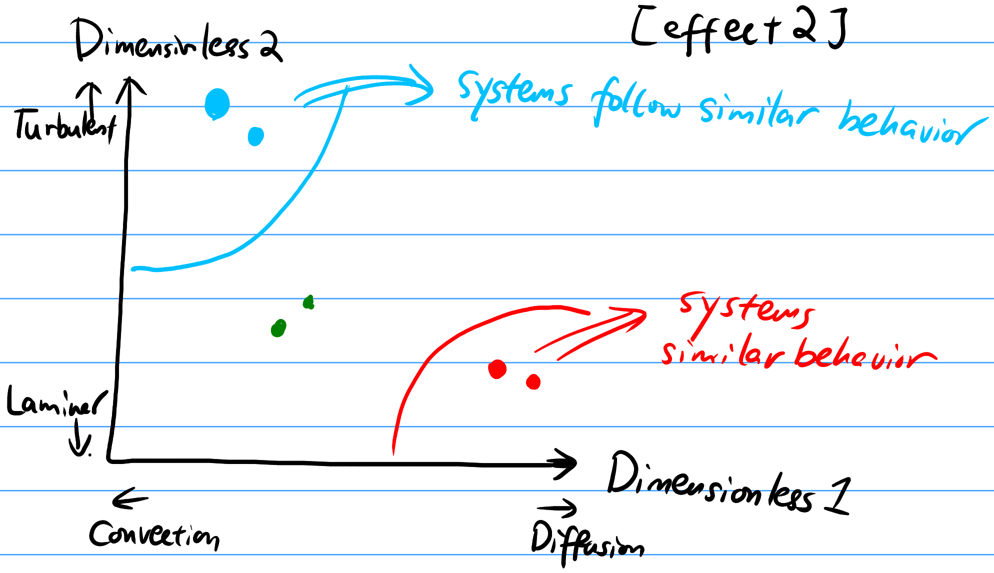
We have seen

1) k_c' ⇒ simple form to $N_A = k_c'(C_{A1} - C_{A2})$
2) k_c' can be taken from theories
   - Film
   - Penetration
   - Boundary layer

What if 1) We're interested in getting accurate k_c'
2) Measured k_c', can we applied to other geometry ?

We can use "Dimensionless Analysis"

$$\text{Dimesionless number} = \frac{[\text{effect } 1]}{[\text{effect } 2]}$$



(μ : viscosity Pa·S)

N_{Sc} (schmidt Nr) $= \dfrac{\mu}{\rho D_{AB}} = \dfrac{[\text{momentum diff}]}{[\text{molar diff}]}$

also $\dfrac{\delta}{\delta_c} = N_{Sc}^{\frac{1}{3}}$

N_{Sh} (Sherwood Nr) $= \dfrac{k_c' L}{D_{AB}} = \dfrac{[\text{conv m.t.}]}{[\text{diff m.t.}]}$

N_{Re} (Reynolds Nr) $= \dfrac{v \rho L}{\mu} = \dfrac{v L}{\left(\frac{\mu}{\rho}\right)} = \dfrac{[\text{kinetic transp}]}{[\text{viscosity transp}]}$

How do we correlate k_c' ?

Chilton-Colburn j-factor
(same factor for momentum / heat / mass transfer)

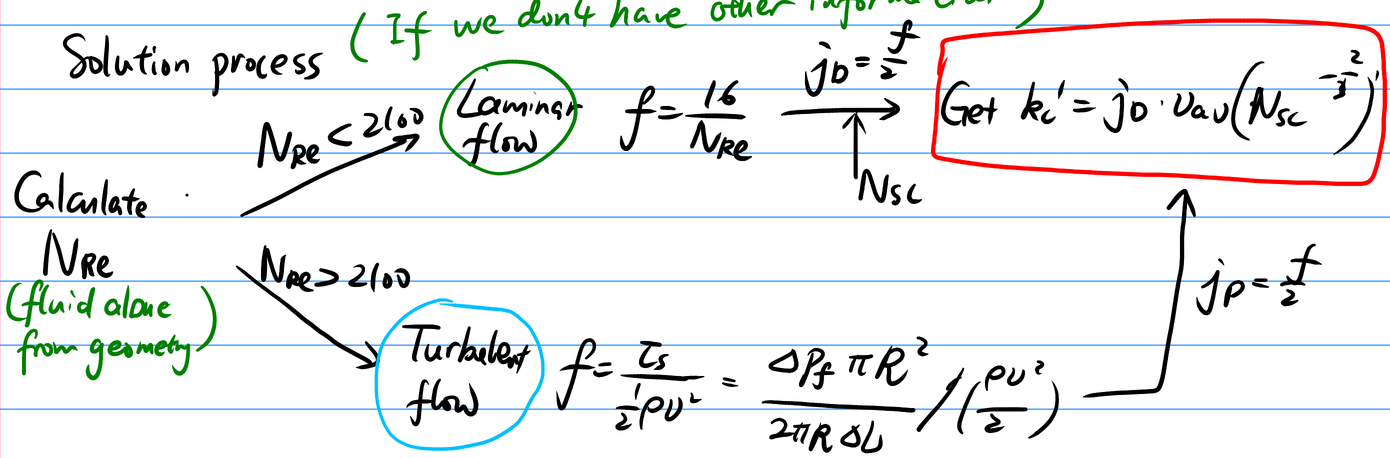
For mass transfer

$$j_D = \frac{k_c'}{v_{av}} (N_{Sc})^{\frac{2}{3}} = \frac{f}{2}$$

f ⇒ Fanning friction factor

v_{av} ↑ average fl. velocity

Solution process (If we don't have other information")



Calculate N_{Re} (fluid alone from geometry)

$N_{Re} < 2100$ → Laminar flow → $f = \dfrac{16}{N_{Re}}$ $\xrightarrow{j_D = \frac{f}{2}}$ (↑ N_{Sc}) → Get $k_c' = j_D \cdot v_{av} \left(N_{Sc}^{-\frac{2}{3}}\right)$

$N_{Re} > 2100$ → Turbulent flow → $f = \dfrac{\tau_s}{\frac{1}{2}\rho v^2} = \dfrac{\Delta P_f \pi R^2}{2\pi R \Delta L} \Big/ \left(\dfrac{\rho v^2}{2}\right)$ $\xrightarrow{j_D = \frac{f}{2}}$ Get k_c'

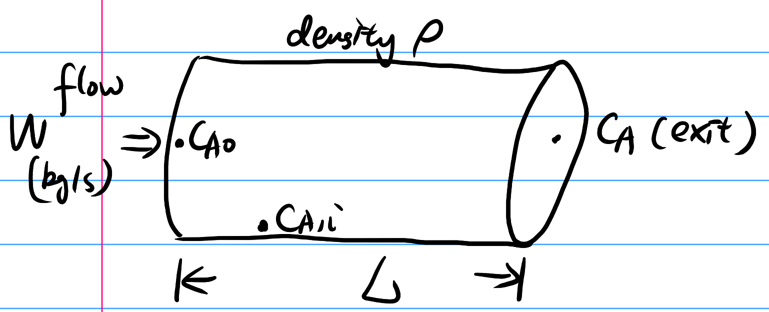
More generally

$$\boxed{j_D = \frac{f}{2} = \frac{N_{Sh}}{N_{Re} N_{Sc}^{\frac{1}{3}}}}$$

N_{Sh} → has k_c'

Most likely, we use dimensionless analysis on an established chart

Pipe



$\frac{W}{D_{AB}\rho L}$ determines exit concentration

$$\frac{\cancel{kg}/\cancel{s}}{\frac{m^2}{\cancel{s}}\cdot\frac{\cancel{kg}}{m^3}\cdot m}$$

$$\frac{W}{D_{AB}\rho L} = N_{Re}N_{Sc}\cdot\frac{D}{L}\cdot\frac{\pi}{4}$$

1) Gas (use rod-lik flow curve)

2) Liquid $\frac{W}{D_{AB}\rho L} > 400$ parabolic flow

$$\boxed{\frac{C_A - C_{A,0}}{C_{A,i} - C_{A,0}} = 5.5\left(\frac{W}{D_{AB}\rho L}\right)^{-2/3}}$$

3) Liquid/Gas turbulent flow

$N_{Re} > 2100$ If $0.6 < N_{Sc} < 3000$

$$N_{Sh} = k_c'\left(\frac{\overset{\text{diamet}}{D}}{D_{AB}}\right) = 0.023\underbrace{\left(\frac{\rho D v}{\mu}\right)}_{N_{Re,D}}^{0.83}\underbrace{\left(\frac{\mu}{\rho D_{AB}}\right)}_{N_{Sc}}^{0.33}$$

1) (very close to the j-D analog)

2) We just need $N_{Re,D}$ N_{Sc} $\Rightarrow k_c'$

3) Only applicable for pipe ! (& N_{Sc} range)

Solution procedure