CHE 318 Lecture 20

Case Studies With Mass Transfer Coefficient (II)

Dr. Tian Tian

2026-02-27

Learning outcomes

After this lecture, you will be able to:

- **Recall** characteristic features of convective mass transfer over flat plates, spheres, and packed beds.
- **Identify** suitable empirical correlations for each geometry and flow regime.
- **Apply** equations to calculate k_c', concentration changes, and fluxes in representative case studies.

Cheatsheet for mass transfer coefficient

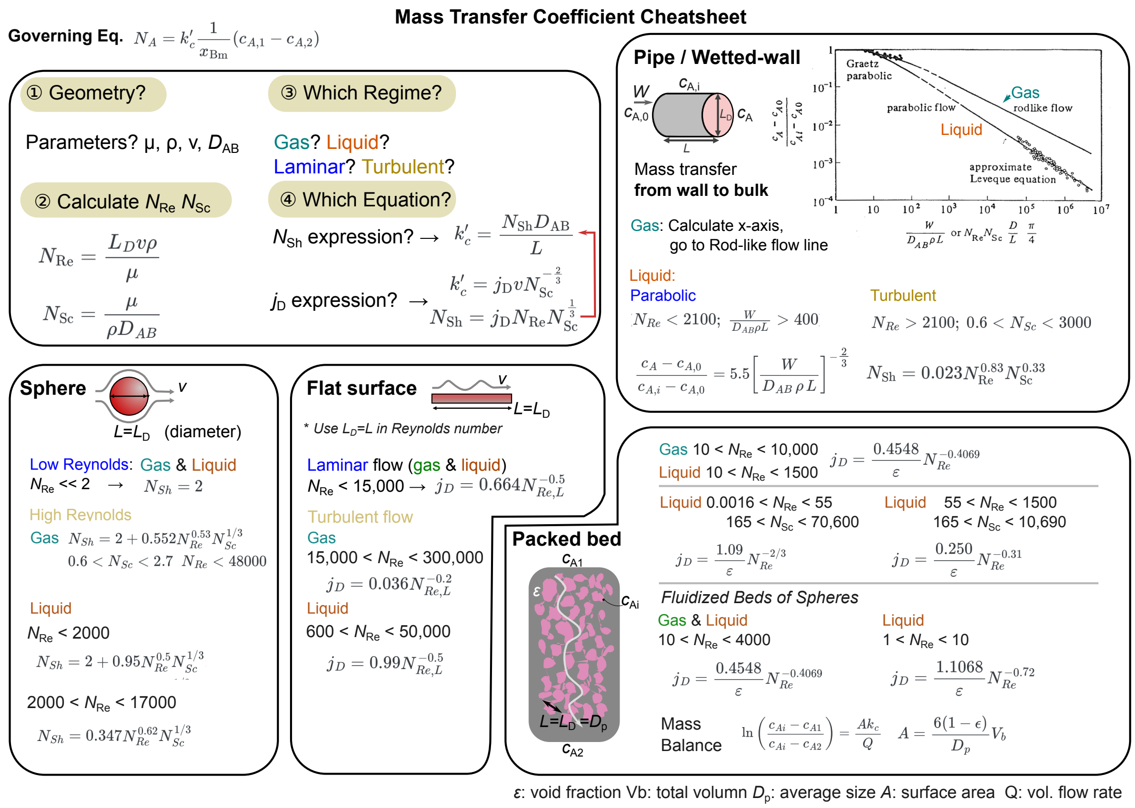

Figure 1: Cheatsheet for using dimensionless numbers with k_c'. Printed version distributed in class

Case 2: flow parallel to flat plates

- Can be used for gases or evaporation of liquid
- Distinguished between laminar & turbulent flow
- N_{Re} regime cutoff different in gas & liquid!
- Characteristic length L: length of plate in flow direction
- Flow parallel to flat plate: video

Flow past parallel plates: regimes

Laminar flow ($N_{Re} < 15,000$)

$$j_D = 0.664 N_{Re,L}^{-0.5} \tag{1}$$

$$\frac{k_c' L}{D_{AB}} = 0.664 N_{Re,L}^{0.5} N_{Sc}^{1/3} \tag{2}$$

- This follows our derivation of boundary layer theory

Turbulent flow

- Gas: $15,000 < N_{Re} < 300,000$

$$j_D = 0.036 N_{Re,L}^{-0.2} \tag{3}$$

- Liquid: $600 < N_{Re} < 50,000$

$$j_D = 0.99 N_{Re,L}^{-0.5} \tag{4}$$

Example for flat plate

Example 7.3-2: A large volume of pure water at 26.1 °C is flowing parallel to a flat plate of solid benzoic acid, where $L = 0.244$ m in the direction of flow. The water velocity is 0.061 m/s. The solubility of benzoic acid in water is 0.02948 kg mol/m^3, and the diffusivity of benzoic acid in water is 1.245×10^{-9} m^2/s. For water, $\mu = 8.71 \times 10{-}4$ Pa·s and $\rho = 996$ kg/m^3 (basically the same conditions as in case 1-2).

Calculate the mass-transfer coefficient k_c and N_A.

Flat surface: solution steps

- Step 1: calculate N_{Re}, N_{Sc}. Regime?
- Step 2: use flat-surface equations to calculate j_D
- Step 3: convert j_D to k_c'. Use $k_c = k_c'/x_{BM}$
- Step 4: use master equation to get N_A

Flat surface example: results

> **Tip**
>
> We can use $x_{BM} = 1$ in this case

- $N_{\mathrm{Re}} = 1.700 \times 10^4$, $N_{\mathrm{Sc}} = 702$
- Use **liquid turbulent flow**: $j_D = 0.99 N_{\mathrm{Re}}^{-0.5} = 0.00758$
- $k'_c = j_D v N_{\mathrm{Sc}}^{-2/3} = 5.85 \times 10^{-6}$ m/s
- $N_A = 1.726 \times 10^{-7}$ kg mol/m^2/s

Case 3: flow past single sphere

- Frequent geometry in particle solutions
- Low Reynolds regime   solution for stagnant diffusion on spherical surface
- High Reynolds regime   correct N_{Sh} and back calculate k'_c

Flow past single sphere: results

Low Reynolds ($N_{Re} < 2$)

$$
\begin{aligned}
N_A &= \boxed{\frac{2D_{AB}}{D_p}}(c_{A1} - c_{A2}) && (5)\\
&= k_c(c_{A1} - c_{A2}) && (6)\\
&= \frac{k'_c}{x_{Bm}}(c_{A1} - c_{A2}) && (7)\\
& && (8)
\end{aligned}
$$

- For $x_{Bm} \approx 1$, we have:

$$
k'_c = \frac{2D_{AB}}{D_p}
$$

- Sherwood number: $N_{Sh} = 2$

High Reynolds ($N_{Re} > 2$)

- Gas:

$$N_{Sh} = 2 + 0.552 N_{Re}^{0.53} N_{Sc}^{1/3} \tag{9}$$

$$0.6 < N_{Sc} < 2.7 \quad N_{Re} < 48000 \tag{10}$$

- Liquid:

$$N_{Sh} = 2 + 0.95 N_{Re}^{0.5} N_{Sc}^{1/3};\ N_{Re} < 2000 \tag{11}$$

$$N_{Sh} = 0.347 N_{Re}^{0.62} N_{Sc}^{1/3};\ 2000 < N_{Re} < 17000 \tag{12}$$

- Back calculate $k'_c = N_{Sh} \frac{D_{AB}}{D_p}$

Case 4: mass transfer for packed beds

- Video for fluidized packed bed

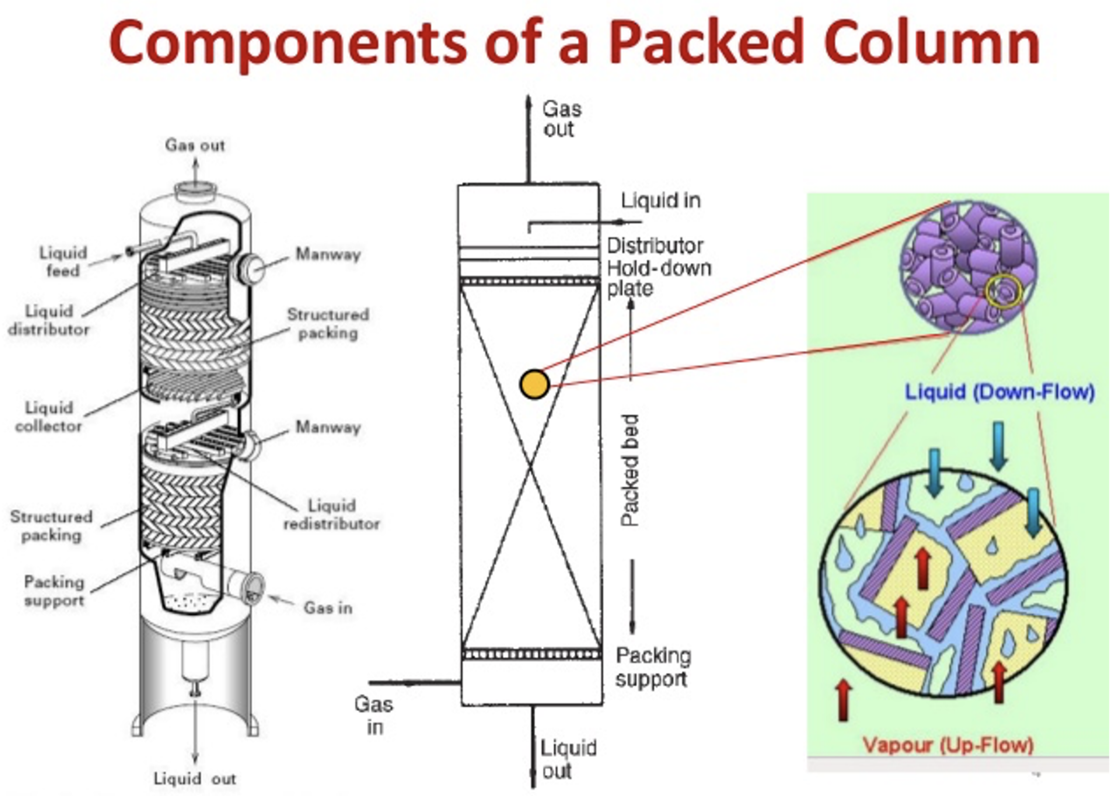

Figure 2: Packed bed structure

Packed bed: geometries

- Geometry characteristics: void fraction ε:

$$\varepsilon = \frac{\text{void space}}{\text{total space}} = \frac{\text{void space}}{\text{void space} + \text{solid space}}$$

  - Typically $0.3 < \varepsilon < 0.5$
  - Void fraction is difficult to measure experimentally

- Total Effective Area for spherical particles:

$$A = \frac{6(1-\epsilon)}{D_p} V_b$$

Correlation equations in packed bed

Correlation 1, applicable to:

- gase with $10 < N_{Re} < 10,000$
- liquid with $10 < N_{Re} < 1500$

$$j_D = j_H = \frac{0.4548}{\varepsilon} N_{Re}^{-0.4069} \tag{13}$$

$$N_{Re} = \frac{D_p v' \rho}{\mu} \tag{14}$$

- D_p: (average) particle diameter
- v': superficial velocity in the tube without packing

Correlation equations in packed bed (II)

Correlation 2, applicable to:

- liquid with $0.0016 < N_{Re} < 55$, $165 < N_{Sc} < 70000$

$$j_D = \frac{1.09}{\varepsilon} N_{Re}^{-2/3} \tag{15}$$

- liquid with $55 < N_{Re} < 1500$, $165 < N_{Sc} < 10690$

$$j_D = \frac{0.250}{\varepsilon} N_{Re}^{-0.31} \tag{16}$$

Correlation equations in packed bed (III)

Correlation 3, applicable to fluidized beds

- $10 < N_{Re} < 4000$ (gas & liquid)

$$j_D = \frac{0.4548}{\varepsilon} N_{Re}^{-0.4069} \tag{17}$$

- $1 < N_{Re} < 10$ (liquid only)

$$j_D = \frac{1.1068}{\varepsilon} N_{Re}^{-0.72} \tag{18}$$

Example 3: comparison between geometries

Adapted from Problem 7.3-3. Let's estimate the gas-phase mass transfer coefficient k_G (kg mol/(m^2 s Pa)) for mass transfer of water vapour to solids with different shapes. Consider a water vapour (A) in air (B) at 338.6 K and 101.32 Pa flowing through a big duct containing solids with various geometries. The flow velocity is 3.66 m/s. The water vapour concentration is small, so property of air is used ($\mu = 2.03 \times 10^-5$ Pa·s, $\rho = 1.043$ kg/m^3). From the table, $D_{AB} = 2.88 \times 10^{-5}$ m^2/s at 315 K. Compare the values for following geometries, which case?

a) Flow parallel to flat plate with length $L = 2.54$ cm
b) A single sphere with diameter $D = 2.54$ cm
c) Packed beds using spheres of average diameter $D_p = 2.54$ cm and $\epsilon = 0.35$

Example 3: results

> Tip
>
> - Do not forget to correct D_{AB} for temperature!
> - Choose the right N_{Sh} or j_D formula according to N_{Re} and N_{Sc}
> - $k_G \approx k'_G = k'_c/(RT)$

- Flat surface: $k_G = 1.738 \times 10^{-8}$ kg mol/(m^2 s Pa). $N_{\text{Sh}} = 38.03$
- Single sphere: $k_G = 1.984 \times 10^{-8}$ kg mol/(m^2 s Pa). $N_{\text{Sh}} = 43.40$
- Packed bed: $k_G = 7.60 \times 10^{-8}$ kg mol/(m^2 s Pa). $N_{\text{Sh}} = 166.32$

Packed bed clearly wins!

Summary

- Dimensionless numbers can be used to correlate mass transfer problems in different flow rate, dimension etc
- Typically, start with a known geometry (pipe? parallel plate? sphere? packed bed?)
- Find the correlation with dimensionless numbers N_{Re}, N_{Sc}
- Calculate the final mass transfer rate
- Geometry plays an important role in determining k_c'!