CHE 318 Lecture 21

In-Depth Analysis of Packed Bed Columns

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Learning outcomes

After this lecture, you will be able to:

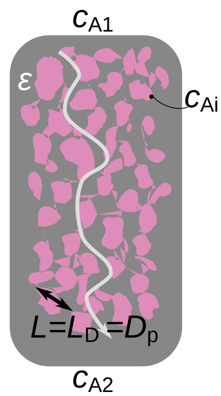
- **Recall** how packed-bed mass transfer coefficients compare with those of other geometries.
- **Describe** mass-balance equations for packed-bed columns.
- **Identify** concentration and pressure profiles inside a packed column.

Recall from last week: mass transfer correlations

- Goal: calculate k'_c from:
  - geometry (L_D, L, D_P, etc)
  - properties (μ, ρ, D_{AB}, v, etc)
- Calculate N_{Re}, N_{Sc}
- Get j_D and / or N_{Sh} to back-calculate k'_c

A deeper look into packed bed

Packed bed



Gas $10 < N_{Re} < 10{,}000$
Liquid $10 < N_{Re} < 1500$

$$j_D = \frac{0.4548}{\varepsilon} N_{Re}^{-0.4069}$$

Liquid $0.0016 < N_{Re} < 55$, $165 < N_{Sc} < 70{,}600$

$$j_D = \frac{1.09}{\varepsilon} N_{Re}^{-2/3}$$

Liquid $55 < N_{Re} < 1500$, $165 < N_{Sc} < 10{,}690$

$$j_D = \frac{0.250}{\varepsilon} N_{Re}^{-0.31}$$

Fluidized Beds of Spheres

Gas & Liquid
$10 < N_{Re} < 4000$

$$j_D = \frac{0.4548}{\varepsilon} N_{Re}^{-0.4069}$$

Liquid
$1 < N_{Re} < 10$

$$j_D = \frac{1.1068}{\varepsilon} N_{Re}^{-0.72}$$

Mass Balance

$$\ln\left(\frac{c_{Ai} - c_{A1}}{c_{Ai} - c_{A2}}\right) = \frac{Ak_c}{Q} \qquad A = \frac{6(1-\epsilon)}{D_p} V_b$$

ε: void fraction Vb: total volumn D_p: average size A: surface area Q: vol. flow rate

Recall from last week: comparison between geometries

Adapted from Problem 7.3-3. Let's estimate the gas-phase mass transfer coefficient k_G (kg mol/(m^2 s Pa)) for mass transfer of water vapour to solids with different shapes. Consider a water vapour (A) in air (B) at 338.6 K and 101.32 Pa flowing through a big duct containing solids with various geometries. The flow velocity is 3.66 m/s. The water vapour concentration is small, so property of air is used ($\mu = 2.03 \times 10^{-}5$ Pa·s, $\rho = 1.043$ kg/m^3). From the table, $D_{AB} = 2.88 \times 10^{-5}$ m^2/s at 315 K. Compare the values for following geometries, which case?

a) Flow parallel to flat plate with length $L = 2.54$ cm
b) A single sphere with diameter $D = 2.54$ cm
c) Packed beds using spheres of average diameter $D_p = 2.54$ cm and $\epsilon = 0.35$

Case 3: results

💡 Tip

- Do not forget to correct D_{AB} for temperature!
- Choose the right N_{Sh} or j_D formula according to N_{Re} and N_{Sc}
- $k_G \approx k'_G = k'_c/(RT)$

- Flat surface: $k_G = 1.738 \times 10^{-8}$ kg mol/(m^2 s Pa). $N_{\text{Sh}} = 38.03$
- Single sphere: $k_G = 1.984 \times 10^{-8}$ kg mol/(m^2 s Pa). $N_{\text{Sh}} = 43.40$
- Packed bed: $k_G = 7.60 \times 10^{-8}$ kg mol/(m^2 s Pa). $N_{\text{Sh}} = 166.32$

Packed bed clearly wins!

Why packed bed columns have better mass transfer?

- Packed bed geometry provides large surface-to-volume ratio (ϵ and D_p)
- The boundary layer length δc is small due to continuous disruption of liquid/gas interface
- Overall k'_c is larger than single geometry
- Pressure drop Δp over the column can be larger than other types!

Compute mass transfer in packed bed columns

In a packed bed design, it is often desired to know the following quantities:

1) The outlet concentration c_{A2}
2) The total flux of mass transfer across interface

- It is still desired to use [In] - [Out] + [Gen] = [Acc]
- What are each of these quantities?

Mass balance in packed beds

- Volumetric flow rate is Q (unit m^3/s)
- Inlet, outlet concentration: c_{A1}, c_{A2}
- Cross sectional area S
- Interfacial mass transfer flux N_A, effective area A_{eff}

$$[\text{In}] - [\text{Out}] + [\text{Gen}] = [\text{Acc}] \tag{1}$$

$$Q(c_{A1} - c_{A2}) + A_{\text{eff}}\hat{N}_A = 0 \tag{2}$$

The effective area A_{eff}

- A_{eff} related to the surface-to-volume ratio a and total bed volume V_b

$$A_{\text{eff}} = aV_b \tag{3}$$

- For spheres, we can derive a (unit m^2/m^3)

$$a = \frac{6(1-\epsilon)}{D_p} \tag{4}$$

How to get $\hat{N}_A$?

- $\hat{N}_A$ is the **average** mass transfer flux across the interface.
- To solve it, we still use a control volume from z to $z + dz$
- Locally, driving force $c_{Ai} - c_A(z)$
- Local mass transfer flux

$$N_A(z) = k_c\left[c_{Ai} - c_A(z)\right] \tag{5}$$

Differential equation for $c_A(z)$

$$N_A a S dz = Q dc_A \tag{6}$$

$$\frac{k_c a S}{Q} dz = \frac{dc_A}{c_{Ai} - c_A(z)} \tag{7}$$

Integrating over z we get:

$$\ln\left(\frac{c_{Ai} - c_{A1}}{c_{Ai} - c_A(z)}\right) = \frac{k_c a S z}{Q} \tag{8}$$

Concentration profile of packed bed (single phase)

- When c_{Ai} is constant (e.g. solid-gas interface), concentration follows:

$$c_A(z) = c_{Ai} - (c_{Ai} - c_{A1}) \exp(-\frac{k_c a S}{Q} z) \tag{9}$$

- Analog: reactive wall in pipe system (unsteady-state mass transfer in Lecture 11)

$$c_A(z) = c_{Ai} - (c_{Ai} - c_{A1}) \exp(-\frac{4 k_c z}{D v_m}) \tag{10}$$

The pipe wall problem corresponds to $a = 4/D$, similar to a packed bed problem.

- Clearly, packed bed can achieve saturation much faster, because D_p is usually a few mm to 1 inch.

Outlet concentration for packed bed

- If a packed bed column has height H, outlet concentration c_{A2} follows:

$$c_{A2} = c_{Ai} - (c_{Ai} - c_{A1}) \exp(-\frac{k_c a S H}{Q}) \tag{11}$$

- Rearrange gives

$$\ln\left(\frac{c_{Ai} - c_{A1}}{c_{Ai} - c_{A2}}\right) = \frac{k_c a S H}{Q} \tag{12}$$

$$= \frac{k_c A_{\text{eff}}}{Q} \tag{13}$$

Solving the average mass transfer flux $\hat{N}_A$

- We can further use the mass balance equation $Q(c_{A1} - c_{A2}) + \hat{N}_A A_{\text{eff}} = 0$.
- Use the fact $c_{A2} - c_{A1} = (c_{Ai} - c_{A1}) - (c_{Ai} - c_{A2})$

That gives us

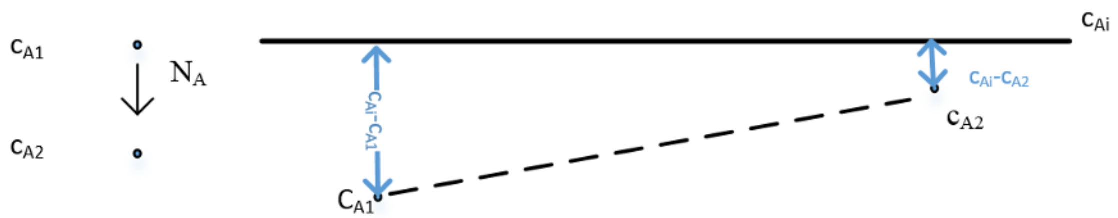
$$\hat{N}_A = k_c \frac{(c_{Ai} - c_{A1}) - (c_{Ai} - c_{A2})}{\ln\left(\frac{c_{Ai} - c_{A1}}{c_{Ai} - c_{A2}}\right)}$$

Average mass transfer flux by log-mean driving force

The previous result means the average driving force in a exponentially-changing concentration profile like in packed bed, should be expressed in **log-mean driving force** form.

It is an expression we will frequently see in other column setups with mixing interfaces

$$\hat{N}_A = k_c \frac{(c_{Ai} - c_{A1}) - (c_{Ai} - c_{A2})}{\ln\left(\frac{c_{Ai} - c_{A1}}{c_{Ai} - c_{A2}}\right)}$$



Example 4: packed bed design

In this lecture, we show that the outlet concentration will be saturated when the tube is long enough.

$$c_{A2} = c_{Ai} - (c_{Ai} - c_{A1}) \exp(-\frac{Ak_c}{Q}) \tag{14}$$

- Consider all parameters in case 3, how high should we design the packed bed to ensure saturated outlet concentration?
- $D_p = 2.54$ cm, $\epsilon = 0.35$, $k_c' = 0.214$ m/s, $v_m = 3.66$ m/s
- You can consider effective saturation means $\frac{Ak_c}{Q} = 5$

Example 4: packed bed design – results

- Minimal height: $H_{\min}$

$$H_{\min} = \frac{5 v_m D_p}{6(1-\epsilon) k_c'}$$

- In case 3 setup, $H_{\min} = 0.556$ m

Summary

- Dimensionless numbers can be used to correlate mass transfer problems in different flow rate, dimension etc
- Typically, start with a known geometry (pipe? parallel plate? sphere? packed bed?)
- Find the correlation with dimensionless numbers N_{Re}, N_{Sc}
- Calculate the final mass transfer rate