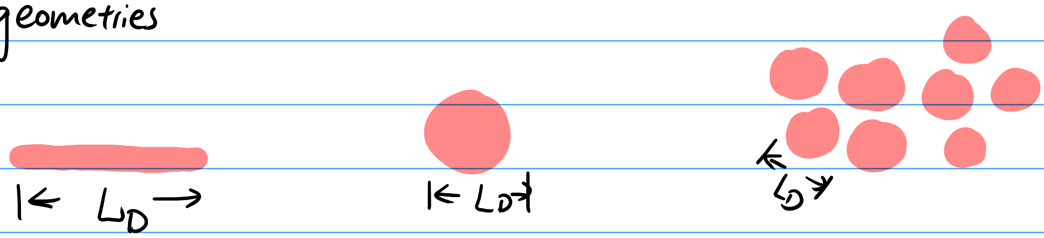
CHE 318 L21

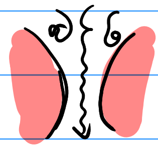
Analysis of Packed Bed

In last lecture, we have compared k_c' in various geometries



Even the same L_D $\Rightarrow$ packed bed wins, why?

1) Inter-particle flow has enhanced convection



2) For liquidized bed (particles move), more frequent interfacial renewal

3) Generally speaking the boundary layer theory $k_c \propto \frac{1}{\delta_c}$

δ_c in packed bed is short (qualitatively)

4) active area of contact is large!

Packed bed mass balance

$$\text{In} - \text{Out} + \text{Gen} = \cancel{\text{Acc}}$$

$$Q(C_{A1} - C_{A2}) + \hat{N}_A \cdot A_{eff} = 0$$

$\hat{N}_A$: average N_A over z from $0 \sim H$

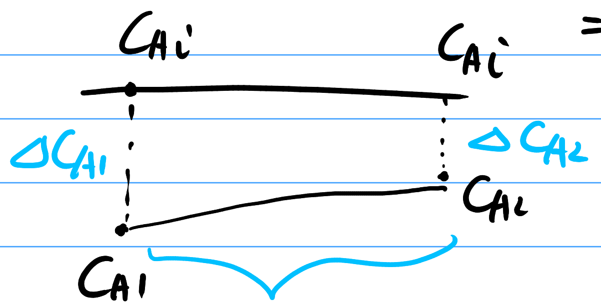
$$A_{eff}\,\hat{N}_A = Q \cdot (C_{A2} - C_{A1})$$

$$A_{eff} \leftarrow \frac{6(1-\varepsilon)}{D_p} \cdot V_b$$

$\hat{N}_A$?

$$N_A = k_c \frac{(C_{Ai} - C_{A1}) - (C_{Ai} - C_{A2})}{\ln\left(\frac{C_{Ai} - C_{A1}}{C_{Ai} - C_{A2}}\right)}$$

$$= k_c \cdot \text{LogMean}\left[(C_{Ai} - C_{A1}), (C_{Ai} - C_{A2})\right]$$



Driving force is logmean of ΔC_{A1} & ΔC_{A2}

Why $a = \dfrac{6(1-\varepsilon)}{D_p}$ for spheres ?

$$a = \frac{[\text{surface}]}{[\text{volume}]} = \frac{[\text{Surface of 1 sphere}] \times [N^{\#}\ \text{sphere}]}{[\text{volume}]}$$

$$= \frac{4\pi\left(\frac{D_p}{2}\right)^2 \cdot (1-\varepsilon) \cdot V_b \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{D_p}{2}\right)^3}}{V_b}$$

$$= \frac{6(1-\varepsilon)}{D_p}$$

Alternative form for packed bed

$$A \cdot k_c \frac{(C_{Ai} - C_{A1}) - (C_{Ai} - C_{A2})}{\ln\left(\frac{C_{Ai} - C_{A1}}{C_{Ai} - C_{A2}}\right)}$$

$$= A \cdot k_c \frac{\cancel{C_{A2} - C_{A1}}}{\ln\left(\frac{C_{Ai} - C_{A1}}{C_{Ai} - C_{A2}}\right)}$$

$$= Q\,\cancel{(C_{A2} - C_{A1})}$$

$$\ln\left(\frac{C_{Ai} - C_{A1}}{C_{Ai} - C_{A2}}\right) = \frac{A k_c}{Q}$$

$$\frac{C_{Ai} - C_{A1}}{C_{Ai} - C_{A2}} = \exp\left(\frac{A k_c}{Q}\right)$$

$$C_{A2} = C_{Ai} - (C_{Ai} - C_{A1}) \cdot \exp\left(-\frac{A k_c}{Q}\right)$$

analog: reactive wall

$$C_{A2} = C_{Ai} - (C_{Ai} - C_{A1}) \cdot \exp\left(-\frac{4 k_c}{v_m} \cdot \frac{L}{D}\right)$$

$$Q \doteq v_m \cdot \frac{\pi D^2}{4}$$

$$A = \frac{6(1-\varepsilon)}{D_p} \cdot V_b$$

$$\frac{A}{Q} = \frac{4 a \cdot V_b}{\pi v_m D^2} = \frac{4 a \pi D^2 L}{\pi v_m D^2}$$

If a tube $V_b = \pi D^2 \cdot L$

so $$\frac{A}{Q} = \frac{4 a \cdot D}{v_m} \cdot \frac{L}{D}$$

only differ by factor $a \cdot D$

How long do we need the packed tower to be?

For saturation, let $\frac{A \cdot k_c}{Q} = 5$ (exp part is $\exp(-5)$
≈ 0.006)

Previous example

$k_c = 0.2140$ m/s

$$A = \frac{6(1-\varepsilon)}{D_p} \cdot \frac{\pi D^2}{4} \cdot H$$

$$Q = v_m \cdot \frac{\pi D^2}{4}$$

$$\frac{A \cdot k_c}{Q} = \frac{6(1-\varepsilon)}{D_p} \cdot \frac{H \cdot k_c}{v_m} = 5$$

$\Rightarrow 3.66$ m/s

$\Rightarrow 0.0254$

$$H = \frac{5 \cdot v_m \cdot D_p}{6(1-\varepsilon)\, k_c} = \qquad = 0.556\,m$$

$\Rightarrow 0.2140$ m/s

$\Rightarrow 0.35$