CHE 318 Lecture 23

Mass Transfer In Two-Phase Column: Operating Line

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Learning outcomes

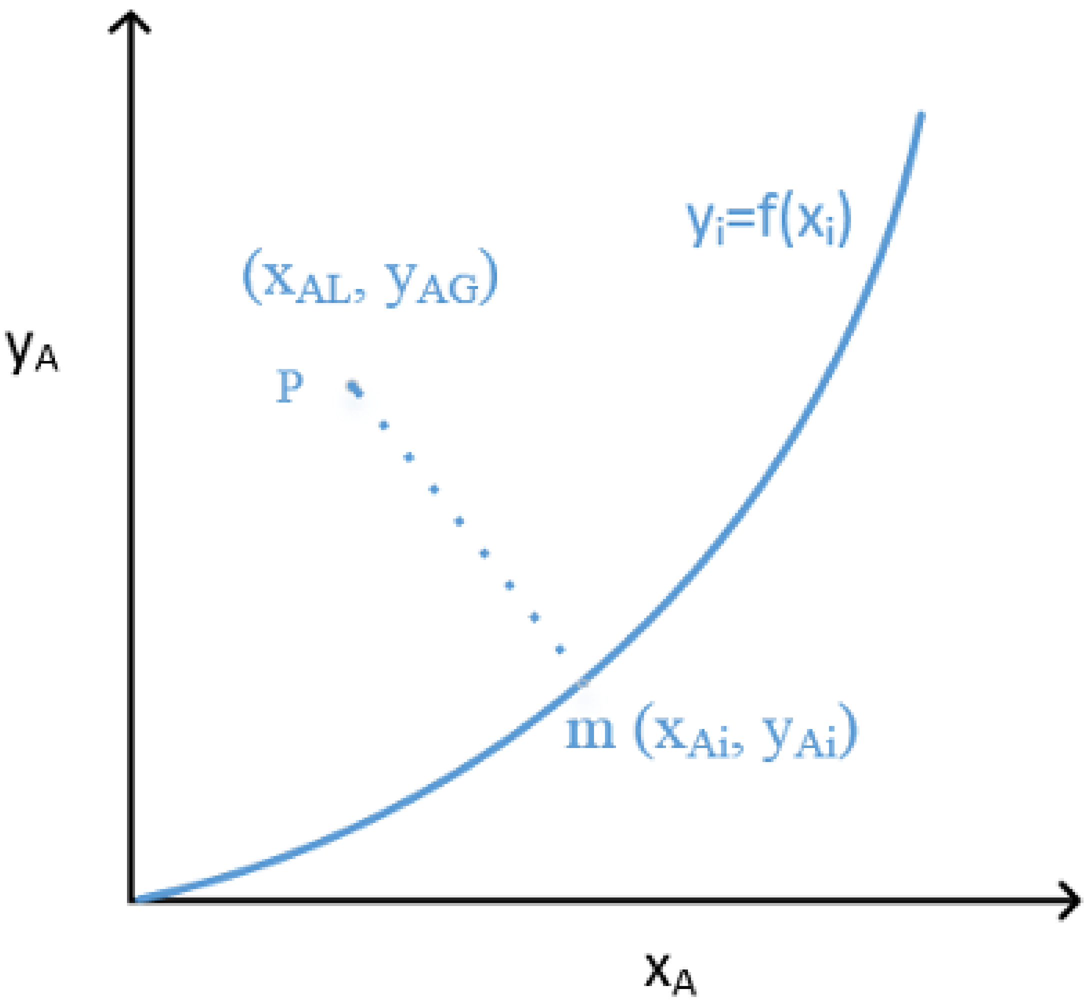
After this lecture, you will be able to:

- **Recall** the overall mass-balance framework for two-phase absorption systems.
- **Identify** the operating line on an equilibrium diagram.
- **Apply** equations to calculate outlet composition and minimum flow rate for an absorption tower.

Recall: reading an equilibrium diagram

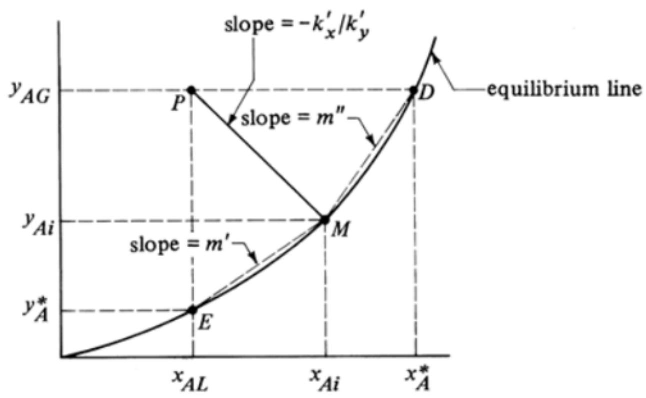
Key features:

- x-axis & y-axis meaning?
- Points on the eq. curve?
- Points above and below eq. curve?
- How to get interfacial composition?
- Slope of line to interfacial points?



Local overall mass transfer coefficients

- Knowing k_x and k_y (film coefficients) allows us to calculate the interfacial concentration (x_{Ai}, y_{Ai})
- **However**, in many industrial applications exact k_x and k_y are hard to find
- Use of overall mass transfer coefficients



Relation between overall and exact mass transfer coefficients

- Exact coefficient k_x, k_y: driving force $y_{AG} - y_{Ai}$

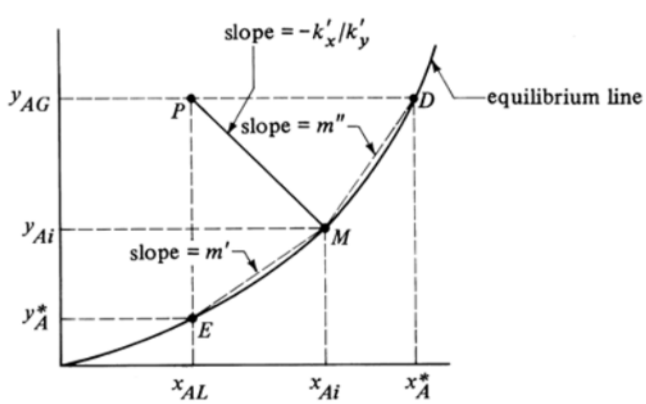
$$N_A = k_y(y_{AG} - y_{Ai})$$

- Overall coefficient K_x, K_y: driving force $y_{AG} - y_A^*$

$$N_A = K_y(y_{AG} - y_A^*) = K_x(x_A^* - x_{AL})$$

K_x and K_y on a diagram

- What are the driving forces associated with K_x and K_y?

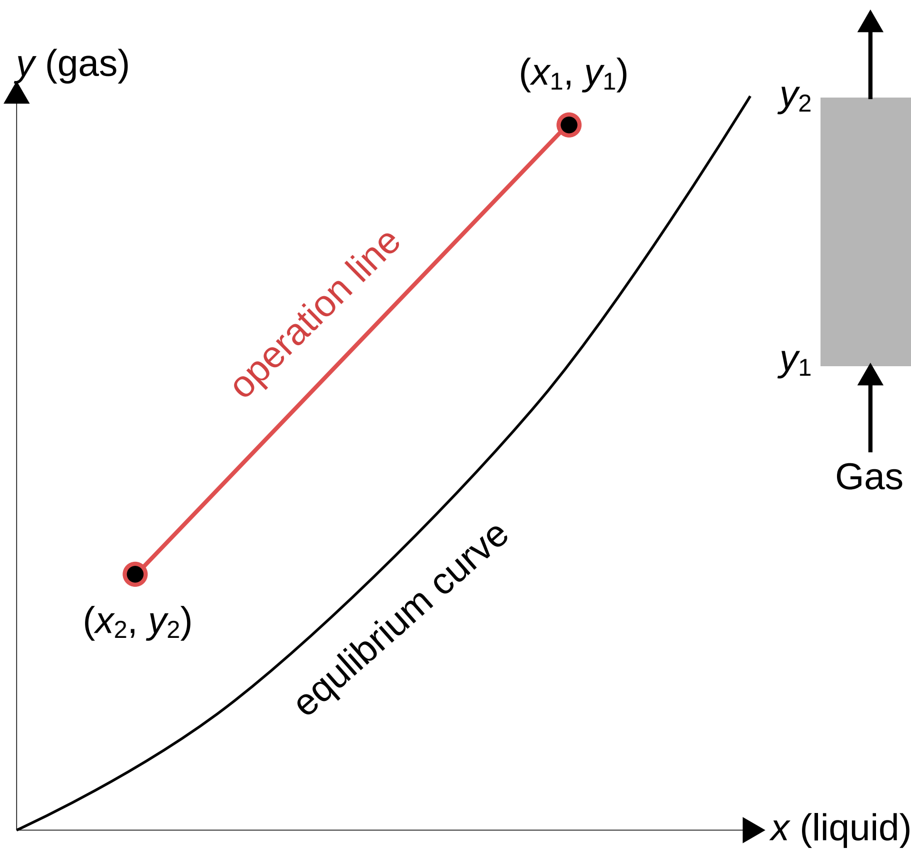


(x, y) relation in a reactor

- Consider an absorption tower with gas inlet at y_{A1} and outlet at y_{A2}
- The gas solute is continuously absorbed by the flowing water phase
- Which is larger, y_{A1} or y_{A2}?

Absorption operating line in equilibrium diagram

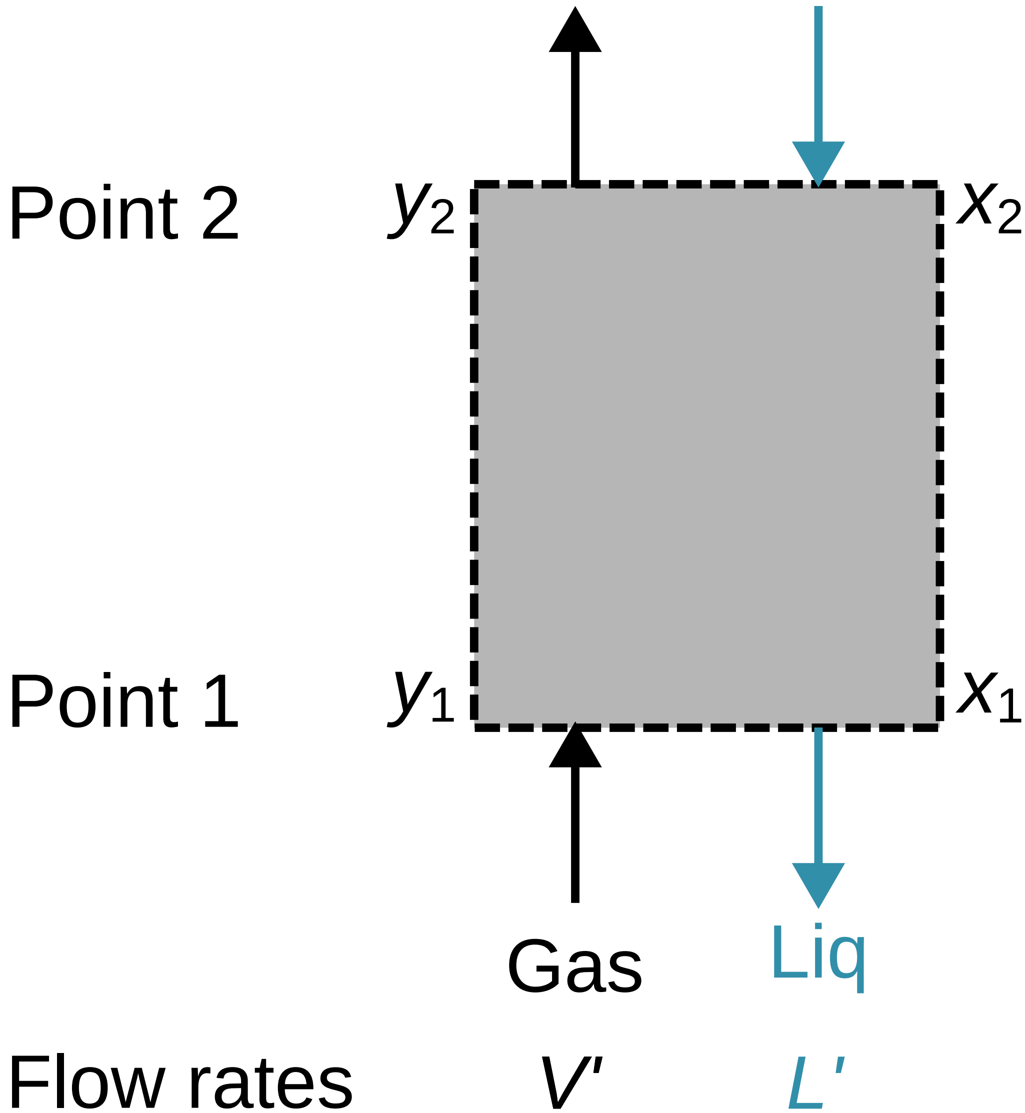
> Tip
>
> - Mass balance tells us $y_{A1} > y_{A2}$
> - A series of (x, y) points during the operation forms the **operating line**



What questions do we want to study?

Given information about the absorption tower, can we answer?

- In- and out-let molar fractions
- Required liquid / gas flow rate
- Concentration profile → **needs to know** N_A
- Height of tower needed → **needs to know** N_A



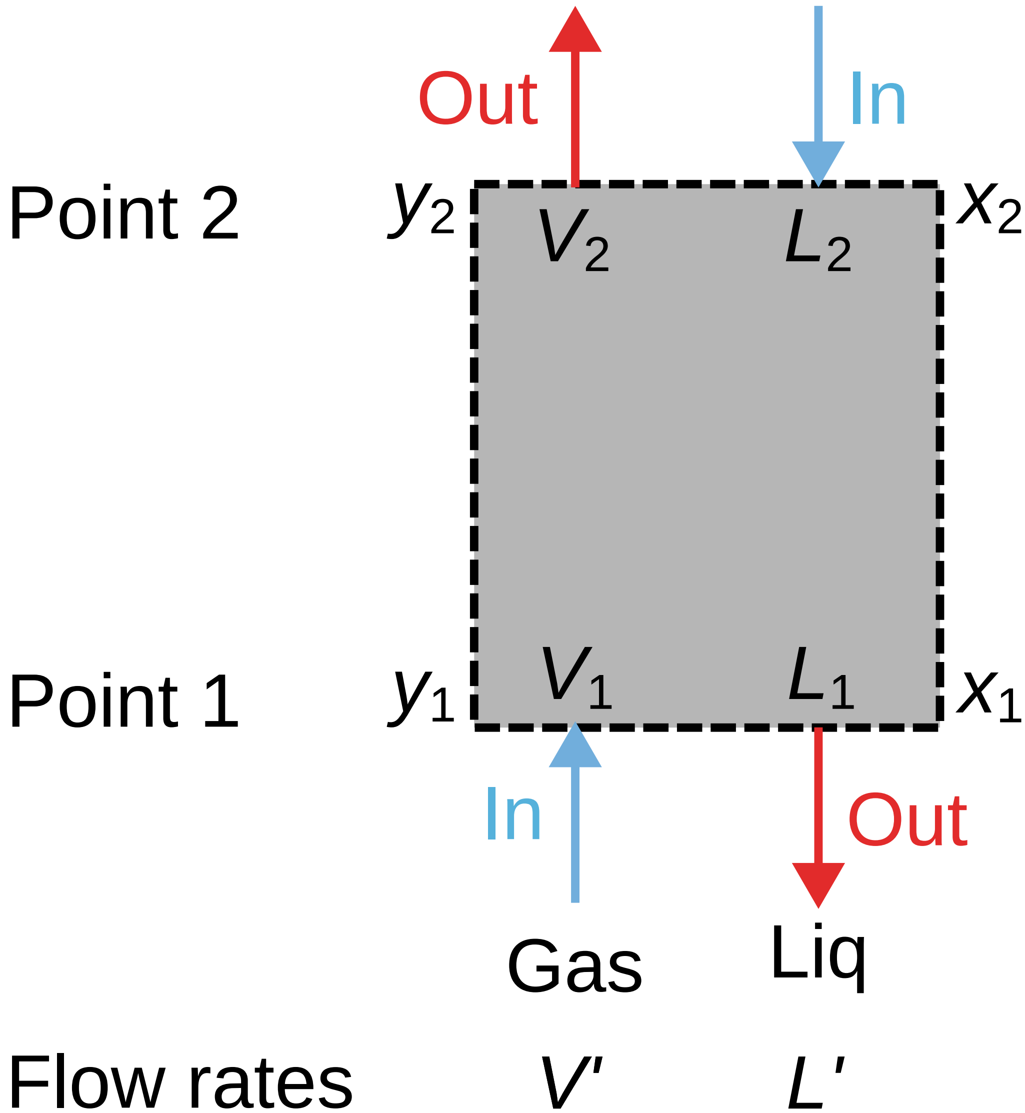
Mass balance in whole control volume

Mass balance for 2 phases:

$$\text{In}_{\text{liq}} + \text{In}_{\text{gas}} = \text{Out}_{\text{liq}} + \text{Out}_{\text{gas}} \tag{1}$$

$$L_2 x_2 + V_1 y_1 = L_1 x_1 + V_2 y_2 \tag{2}$$

- $L_1 = L' + L_{x1}$ (L': flow rate inert liquid)
- $V_1 = V' + V_{y1}$ (V': flow rate inert gas)



Explanation for flow rates

> **Note**
>
> Other flow rates: Q (m^3/s); W (kg/s); v (m/s)

- L: molar flow rate (kg mol/s) for liquid phase
  - L': flow rate for inert liquid
  - L_{x1}: flow rate for A at molar fraction x_1
- V: molar flow rate (kg mol/s) for gas phase
  - V': flow rate for inert gas
  - V_{y1}: flow rate for A at molar fraction y_1

Mass balance for operating line

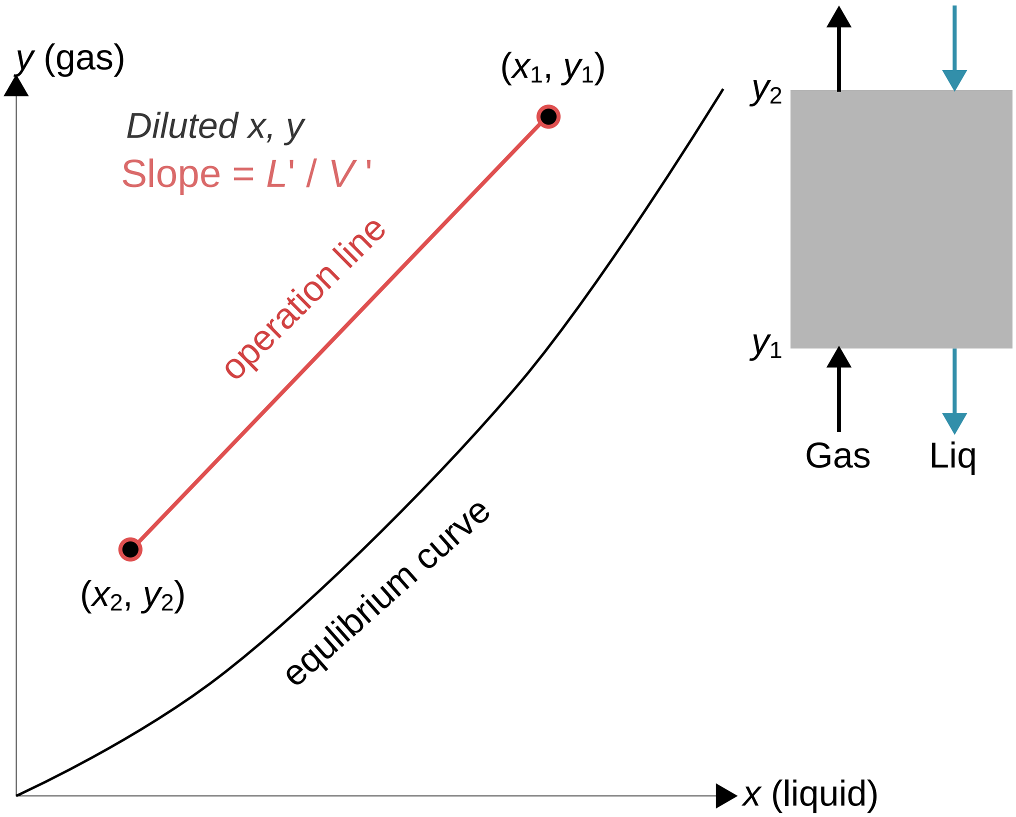
The two ends of the operating line (x_1, y_1) and (x_2, y_2) follow:

$$L'\left(\frac{x_2}{1-x_2}\right) + V'\left(\frac{y_1}{1-y_1}\right) = L'\left(\frac{x_1}{1-x_1}\right) + V'\left(\frac{y_2}{1-y_2}\right) \tag{3}$$

Meaning of the operating line

When $1 - x_1 \approx 1$ and $1 - y_1 \approx 1$, we can rewrite the mass balance equation for any (x, y) along the operating line

$$y = \left(\frac{L'}{V'}\right) x + \left[y_1 - \left(\frac{L'}{V'}\right) x_1\right] \tag{4}$$



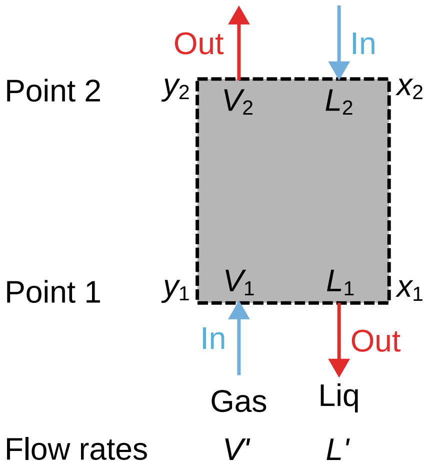
Absorption tower design requirements

In absorption tower, we usually know the following quantities:

- Gas inlet fraction y_1 and flow rate V_1
  - V' can be calculated
- Liquid inlet fraction x_2 (usually $x_2 = 0$)
- The equilibrium curve $y = f_{\text{eq}}(x)$

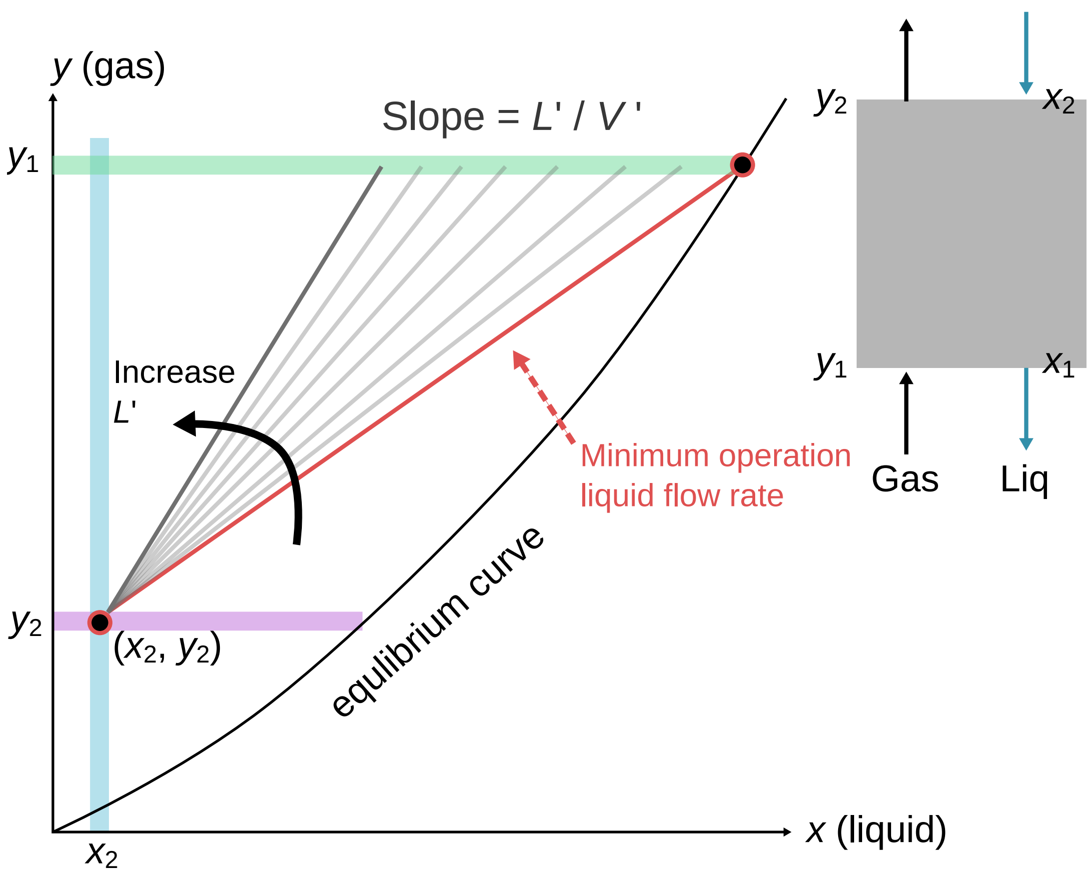
One of the following design goals may be asked:

1. Know the flow rate $L' \rightarrow$ Determine y_2
2. y_2 needs to be at certain value $\rightarrow$ Determine L'



Minimum operating line for absorption tower

For question 2, we know the requirement for y_2, combine with the equilibrium chart, there is a minimum liquid flow rate L'_{min}.



Flow rate in absorption tower

From the dilute regime operating line, the slope is determined by $\left(\frac{L'}{V'}\right)$. Although there is a minimal $\left(\frac{L'}{V'}\right)$ requirement, practical operating line has slope that follows

$$\left(\frac{L'}{V'}\right) \approx 1.5 \times [\text{Slope of Eq. Curve}]$$

- If L'/V' is too high, the column usually needs a larger diameter to compensate the pressure drop.
- If L'/V' is too low, a taller absorption tower is needed for sufficient contact area.

Example 1: determine the outlet composition

A mixture of gas A in air kept at total pressure of 1 atm flows through an absorption tower with flowing water at 293 K. The inlet gas flow rate is $V_1 = 100$ kg mol/h, and inlet $y_1 = 0.20$. The liquid inlet flow rate is $L' = 300$ kg mol/h and inlet contains no dissolved gas ($x_2 = 0$). At the outlets the gas-liquid phases reach equilibrium following the Henry's law:

$$y_2 = mx_1$$

Calculate the outlet mole fraction y_2 and x_1 for the following cases:

1) A is CO_2, $m = 1.42 \times 10^3$
2) A is SO_2, $m \approx 10$

Example 2: solution process

- Write the mass balance equation

$$L'\left(\frac{x_2}{1-x_2}\right) + V'\left(\frac{y_1}{1-y_1}\right) = L'\left(\frac{x_1}{1-x_1}\right) + V'\left(\frac{y_2}{1-y_2}\right) \tag{5}$$

- Obtain L' and V'
- x_1 and y_2 relation from Henry's law

Example 2: results

> ⚠ Warning
>
> y_1 and y_2 are supposedly large, so $1 - y_1 \approx 1$ is not correct!

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CO2 system: x_1 = 1.406e-04, y_2= 0.200
SO2 system: x_1 = 1.592e-02, y_2= 0.159
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Example 2: quanlitative analysis

- Slope of operating line L'/V'

- Slope of equilibrium curve m
- CO_2: $m \gg L'/V'$, can be treated as if gas phase concentration is fixed!
- SO_2: measurable decrease of SO_2 fraction in outlet gas

Summary

- Reading equilibrium chart and operating line
- Mass balance equation for 2 phases
- Solving the flow rate – concentration relation