CHE 318 Lecture 24

Mass Transfer In Two-Phase Column: Realistic Situations

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Learning outcomes

After this lecture, you will be able to:

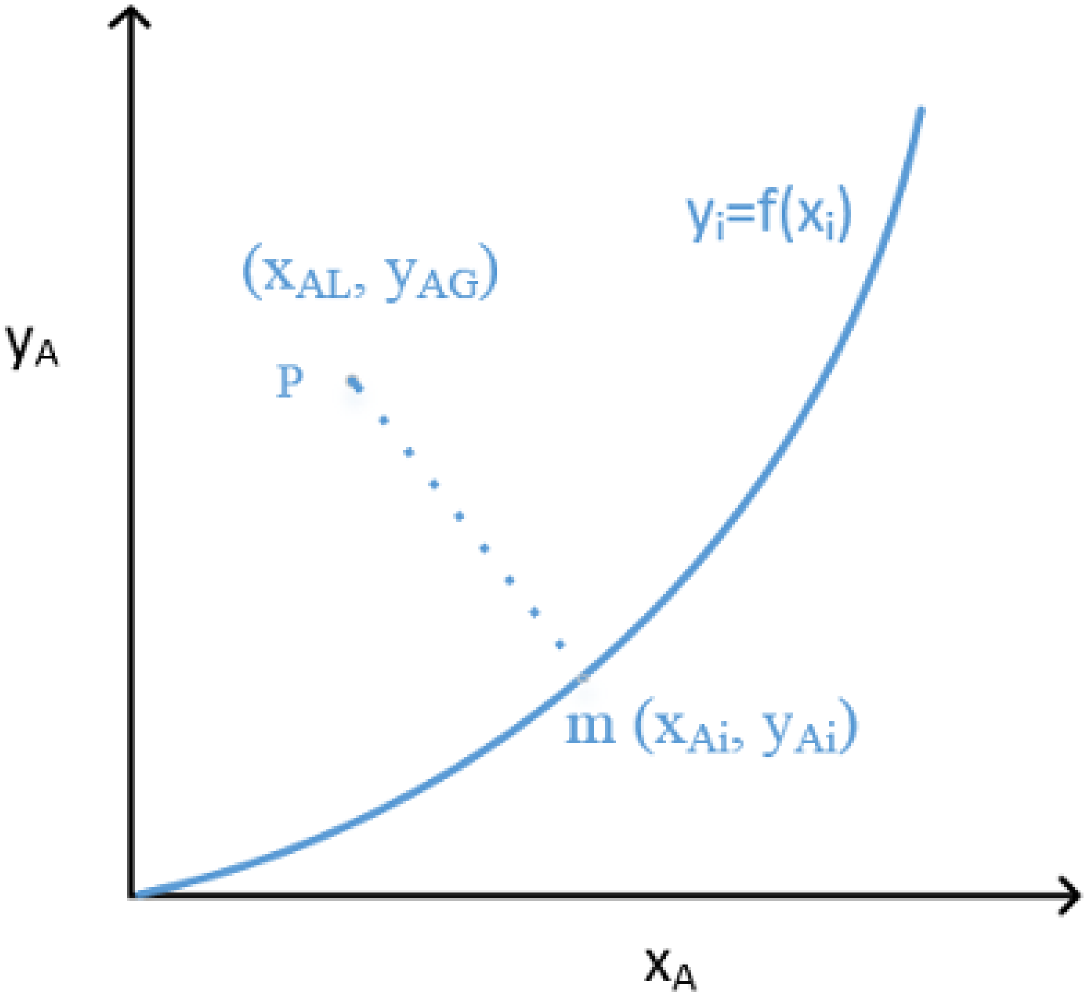
- **Recall** the stagnant-film flux relation for two-phase mass transfer.
- **Apply** film and equilibrium relations to determine interfacial compositions and fluxes.
- **Describe** how internal column mass balances shape local driving forces.

Recall of last week: two-phase mass transfer

Useful tool: equilibrium diagram

Key features:

- x-axis & y-axis meaning?
- Points on and below curve?
- Equilibrium line and operating line?
- Points above and below eq. line?
- Meaning of operating line with flow rates?



Recap: key equation 1 – flux relation

For **very diluted** (EMCD-like) system, the flux equation in each phase follows:

$$N_A = k'_y(y_{AG} - y_{Ai}) = k'_x(x_{Ai} - x_{AL}) \tag{1}$$

This leads to the slope to the eq. line as:

$$\text{Slope} = -\frac{y_{AG} - y_{Ai}}{x_{Ai} - x_{AL}} \tag{2}$$

$$= -\frac{k'_x}{k'_y} \tag{3}$$

Recap: key equation 2 – mass balance (2-phase)

If we only care what possible (x, y) points are in the tower, can use 2-phase mass balance:

$$L'\left(\frac{x_2}{1-x_2}\right) + V'\left(\frac{y_1}{1-y_1}\right) = L'\left(\frac{x_1}{1-x_1}\right) + V'\left(\frac{y_2}{1-y_2}\right) \tag{4}$$

In **very diluted** (EMCD-like) system, we have operating line with a slope of L'/V'.

- Given inlet y_1 and target x_2, y_2 minimal operating L' (see Assignment 6!)
- Given actual L' predict outlet x_1

What can we improve from last week's picture?

Applicability

- Instead of focusing on **very diluted** $(1 - x \approx 1 - y \approx 1)$, derive equations for general, diffusion through stagnant film case
- Solving interfacial composition for general case

Case study: height requirement in packed absorption tower

- How tall should the tower / column be, given the mass transfer coefficients?

More accurate slope for interfacial connection

For non-dilute systems, $1 - x_{AL}$ and $1 - y_{AG}$ may not be close to

- The relation between (x_{AL}, y_{AG}) and (x_{Ai}, y_{Ai}) is no longer linear
- Need correction using log-mean composition terms
- Usual description: diffusion of A through non-diffusing B

Practical relation 1: flux relation for stagnant-film case

Still require the same flux through gas and liquid films:

$$N_A = \frac{k'_y}{(1-y)_{im}}(y_{AG} - y_{Ai}) = \frac{k'_x}{(1-x)_{im}}(x_{Ai} - x_{AL}) \tag{5}$$

So the line connecting bulk point to interface point has slope:

$$\text{Slope} = \frac{y_{AG} - y_{Ai}}{x_{AL} - x_{Ai}} \tag{6}$$

$$= -\frac{k'_x/(1-x)_{im}}{k'_y/(1-y)_{im}} \tag{7}$$

- uses: log mean correction $(1-x)im$ and $(1-y)im$
- depend on the actual location

Note: log mean correction to interfacial flux

- The notations $(1-x)_{im}$ and $(1-y)_{im}$ are log mean values for inert composition between bulk and interface
- Just x_{Bm} in the steady-state diffusion problems
- Will be frequently used in this week's lecture!

$$(1-x)_{im} = x_{Bm} \tag{8}$$

$$= \frac{(1-x_{Ai}) - (1-x_{AL})}{\ln\left(\frac{(1-x_{Ai})}{(1-x_{AL})}\right)} \tag{9}$$

Example 1: finding the interfacial composition (stagnant B)

EXAMPLE 10.4-1. Interface Compositions in Interphase Mass Transfer

The solute *A* is being absorbed from a gas mixture of *A* and *B* in a wetted-wall tower with the liquid flowing as a film downward along the wall. At a certain point in the tower the bulk gas concentration y_{AG} = 0.380 mol fraction and the bulk liquid concentration is x_{AL} = 0.100. The tower is operating at 298 K and 1.013 × 10^5 Pa and the equilibrium data are as follows:

x_A	y_A	x_A	y_A
0	0	0.20	0.131
0.05	0.022	0.25	0.187
0.10	0.052	0.30	0.265
0.15	0.087	0.35	0.385

The solute *A* diffuses through stagnant *B* in the gas phase and then through a nondiffusing liquid.

Using correlations for dilute solutions in wetted-wall towers, the film mass-transfer coefficient for *A* in the gas phase is predicted as k_y = 1.465 × 10^{-3} kg mol A/s · m^2 · mol frac (1.08 lb mol/h · ft^2 · mol frac) and for the liquid phase as k_x = 1.967 × 10^{-3} kg mol A/s · m^2 · mol frac (1.45 lb mol/h · ft^2 · mol frac). Calculate the interface concentrations y_{Ai} and x_{Ai} and the flux N_A.

Solution steps for solving interfacial composition (general case)

Manual trial-and-error steps:

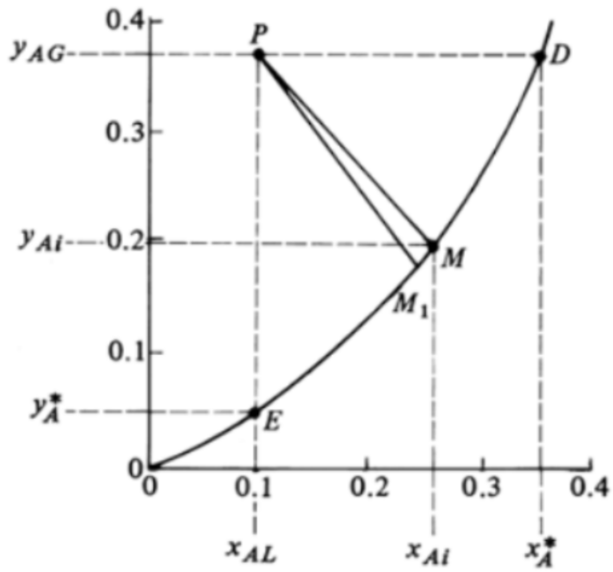
1. Start from bulk point P
2. Guess slope using current correction terms (initially $(1-x)_{im} = (1-y)_{im} = 1$)
3. Connect to equilibrium curve to get (x_{Ai}, y_{Ai})

4. Update $(1-x)im$ and $(1-y)im$, calculate new slope
5. Is new slope in 4) converged?
    - No   go back to step 2
    - Yes   continue to step 6
6. Get final (x_{Ai}, y_{Ai}) and N_A

Example 1: step 1 (follow textbook)

Solution: Since the correlations are for dilute solutions, $(1 - y_A)_{iM}$ and $(1 - x_A)_{iM}$ are approximately 1.0 and the coefficients are the same as k'_y and k'_x. The equilibrium data are plotted in **Fig. 10.4-4**. Point *P* is plotted at $y_{AG} = 0.380$ and $x_{AL} = 0.100$. For the first trial, $(1 - y_A)_{iM}$ and $(1 - x_A)_{iM}$ are assumed as 1.0 and the slope of line *PM* is, from **Eq. (10.4-9)**,

Figure 10.4-4. Location of interface concentrations for Example 10.4-1.



$$-\frac{k'_x/(1 - x_A)_{iM}}{k'_y/(1 - y_A)_{iM}} = -\frac{1.967 \times 10^{-3}/1.0}{1.465 \times 10^{-3}/1.0} = -1.342$$

A line through point *P* with a slope of −1.342 is plotted in **Fig. 10.4-4** intersecting the equilibrium line at M_1, where $y_{Ai} = 0.183$ and $x_{Ai} = 0.247$.

Example 1: step 2 (follow textbook)

A line through point P with a slope of -1.342 is plotted in **Fig. 10.4-4** intersecting the equilibrium line at M_1, where $y_{Ai} = 0.183$ and $x_{Ai} = 0.247$.

For the second trial we use y_{Ai} and x_{Ai} from the first trial to calculate the new slope. Substituting into **Eqs. (10.4-6)** and (**10.4-7**),

$$(1 - y_A)_{iM} = \frac{(1 - y_{Ai}) - (1 - y_{AG})}{\ln[(1 - y_{Ai})/(1 - y_{AG})]}$$

$$= \frac{(1 - 0.183) - (1 - 0.380)}{\ln[(1 - 0.183)/(1 - 0.380)]} = 0.715$$

$$(1 - x_A)_{iM} = \frac{(1 - x_{AL}) - (1 - x_{Ai})}{\ln[(1 - x_{AL})/(1 - x_{Ai})]}$$

$$= \frac{(1 - 0.100) - (1 - 0.247)}{\ln[(1 - 0.100)/(1 - 0.247)]} = 0.825$$

Substituting into **Eq. (10.4-9)** to obtain the new slope,

$$-\frac{k'_x/(1 - x_A)_{iM}}{k'_y/(1 - y_A)_{iM}} = -\frac{1.967 \times 10^{-3}/0.825}{1.465 \times 10^{-3}/0.715} = -1.163$$

A line through point P with a slope of -1.163 is plotted and intersects the equilibrium line at M, where $y_{Ai} = 0.197$ and $x_{Ai} = 0.257$.

Practical relation 2: link overall coefficient to film coefficients

- We learned last week that writing flux equations using K'_x and K'_y are usually easier than k'_x and k'_y
- What is the relation between them?
- Take gas-phase, diffusion through stagnant film

$$N_A = \frac{k'_y}{(1-y)_{im}}(y_{AG} - y_{Ai}) \tag{10}$$

$$= \frac{K'_y}{(1-y)_{*m}}(y_{AG} - y^*_A) \tag{11}$$

$$= K_y(y_{AG} - y^*_A) \tag{12}$$

$(1-y)_{*m}$: log mean between $(1-y_{AG})$ and $(1-y^*_A)$

Geometric interpretation of overall K

Use gas-phase example, from geometry of the equilibrium diagram:

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m'}{k_x} \tag{13}$$

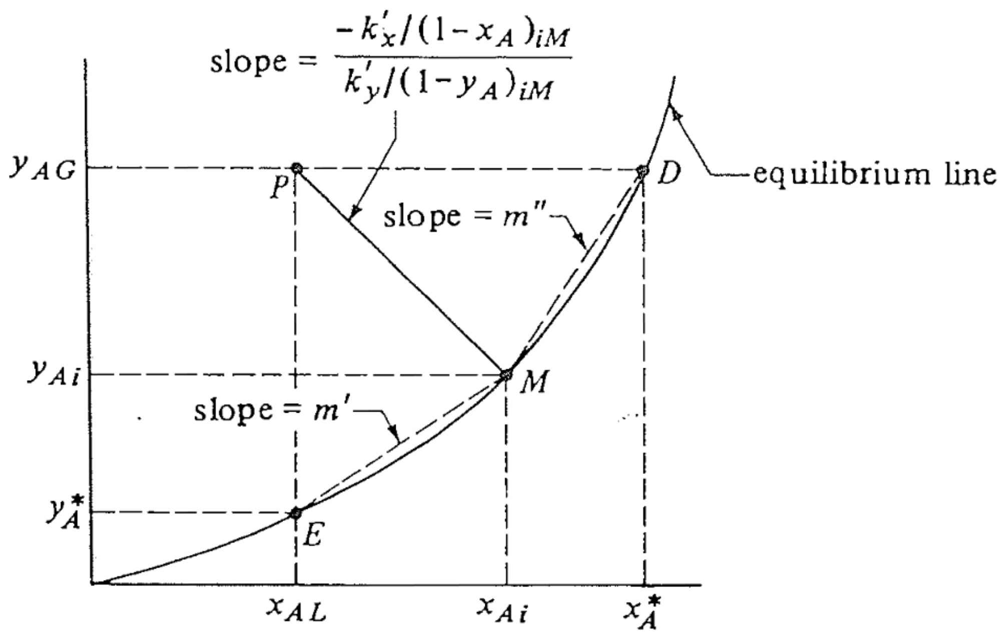


FIGURE 10.4-3. *Concentration driving forces and interface concentrations in interphase mass transfer (A diffusing through stagnant B).*

Transport resistance and overall mass transfer coefficients

- The overall mass transfer coefficient K basically tells which transport resistance is dominant (k inversely proportional to resistance)
- The transport equation becomes a "resistance-in-series" analog

$$[\text{Total resistance in gas}] = \sum \text{resistance in each phase} \tag{14}$$

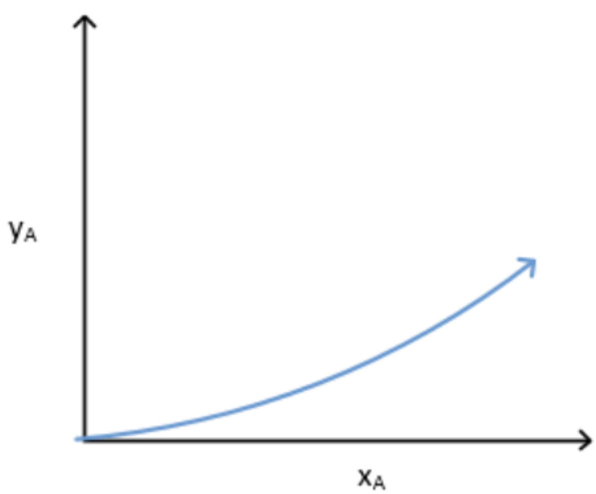
$$\frac{1}{K_y} = \sum f_i \frac{1}{k_i} \tag{15}$$

Case 1: overall K_y in gas phase for highly liquid-soluble A

- Local slope m' is small
- Transfer resistance is mainly in gas!
- Design rule: tune k_y –> more efficient mass transfer

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m'}{k_x} \tag{16}$$

$$\approx \frac{1}{k_y} \tag{17}$$

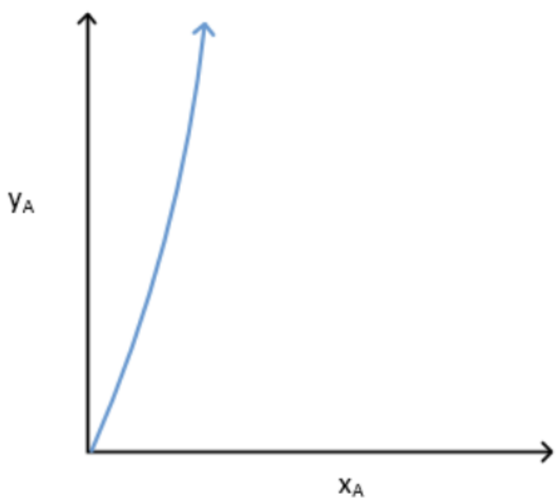


Case 2: overall K_x in liquid phase for low solubility A

- Local slope m'' is large
- Transfer resistance is mainly in liquid!
- Design rule: tune k_x –> more efficient mass transfer

$$\frac{1}{K_x} = \frac{1}{m'' k_y} + \frac{1}{k_x} \tag{18}$$

$$\approx \frac{1}{k_x} \tag{19}$$



Example 2: estimate overall mass transfer coefficients

Use the conditions from example 1: bulk phase point $P = (0.10, 0.380)$, $k'_x = 1.967 \times 10^{-3}$ kg mol/m^2/s and $k'_y = 1.465 \times 10^{-3}$ kg mol/m^2/s, estimate K'_y?

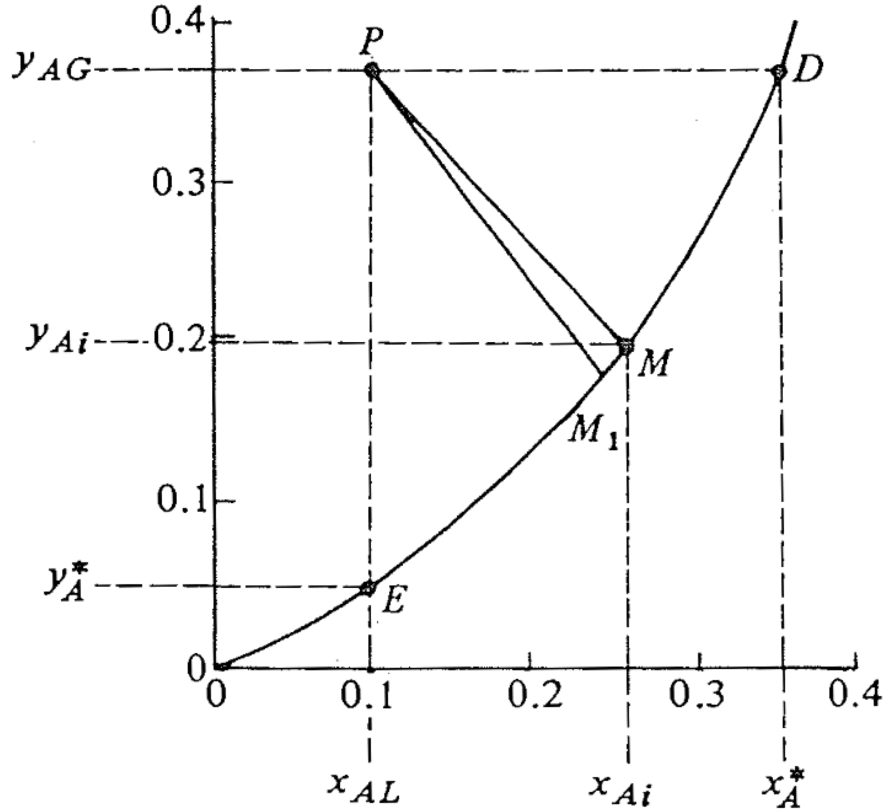


FIGURE 10.4-4. *Location of interface concentrations for Example 10.4-1.*

Example 2: solutions

Solution: From Fig. 10.4-4, $y_A^* = 0.052$, which is in equilibrium with the bulk liquid $x_{AL} = 0.10$. Also, $y_{AG} = 0.380$. The slope of chord m' between E and M from Eq. (10.4-13) is, for $y_{Ai} = 0.197$ and $x_{Ai} = 0.257$,

$$m' = \frac{y_{Ai} - y_A^*}{x_{Ai} - x_{AL}} = \frac{0.197 - 0.052}{0.257 - 0.100} = 0.923$$

From Example 10.4-1,

$$\frac{k'_y}{(1 - y_A)_{iM}} = \frac{1.465 \times 10^{-3}}{0.709} \qquad \frac{k'_x}{(1 - x_A)_{iM}} = \frac{1.967 \times 10^{-3}}{0.820}$$

Using Eq. (10.4-25),

$$(1 - y_A)_{*M} = \frac{(1 - y_A^*) - (1 - y_{AG})}{\ln [(1 - y_A^*)/(1 - y_{AG})]}$$

$$= \frac{(1 - 0.052) - (1 - 0.380)}{\ln [(1 - 0.052)/(1 - 0.380)]} = 0.773$$

Then, using Eq. (10.4-24),

$$\frac{1}{K'_y/0.773} = \frac{1}{1.465 \times 10^{-3}/0.709} + \frac{0.923}{1.967 \times 10^{-3}/0.820}$$

$$= 484.0 + 384.8 = 868.8$$

Solving, $K'_y = 8.90 \times 19^{-4}$. The percent resistance in the gas film is $(484.0/868.8)100 = 55.7\%$ and 44.3% in the liquid film. The flux is as follows, using Eq. (10.4-22):

$$N_A = \frac{K'_y}{(1 - y_A)_{*M}} (y_{AG} - y_A^*) = \frac{8.90 \times 10^{-4}}{0.773} (0.380 - 0.052)$$

$$= 3.78 \times 10^{-4} \text{ kg mol/s} \cdot \text{m}^2$$

This, of course, is the same flux value as was calculated in Example 10.4-1 using the film equations.

Summary

- In-depth analysis of diffusion through stagnant interfacial equilibrium
- Geometric interpretation of equilibrium diagram
- Case studies for interfacial composition & mass transfer coefficient

So far we have built *almost all* prerequisite for solving the concentration profile in the absorption tower! We will discuss that in upcoming Lecture 25