CHE 318 Lecture 25

Mass Transfer In Two-Phase Column: Concentration Profile

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Learning outcomes

After this lecture, you will be able to:

- **Recall** the differential mass-balance framework for two-phase columns.
- **Describe** expressions for packed-column height from local transfer relations.
- **Analyze** coupled gas-liquid concentration profiles along the column.

Cheatsheet for packed bed design

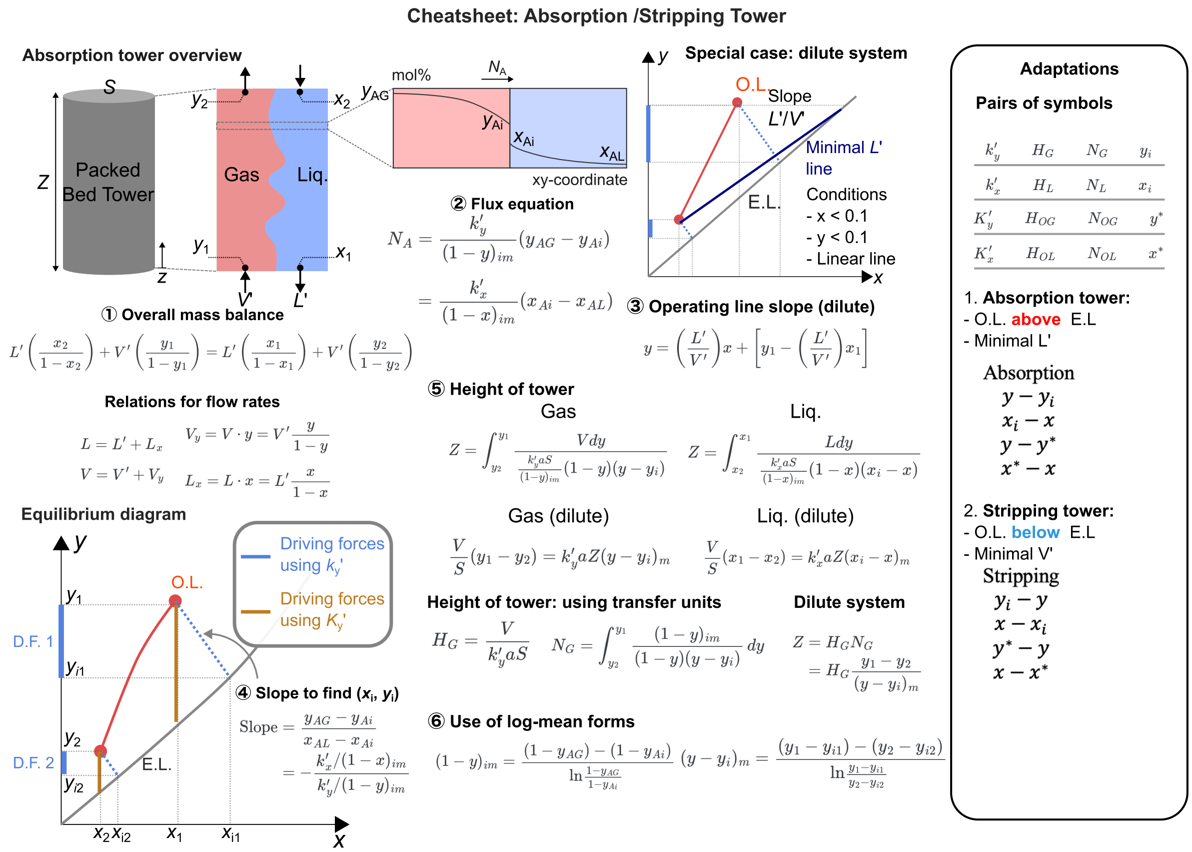


Figure 1: Physical copies distributed in class

Two-phase column: what have we learned so far?

For two-phase mass transfer (solid-state packed bed, liquid-gas absorption tower, etc.), our previous lectures have given:

What we have learned

- Equilibrium diagram
- Operating line (dilute)
- Overall mass balance
- Interfacial composition

What are still missing

- Concentration profile

- Height requirement
- Non-linear operating line

In-depth analysis for packed absorption tower

Example goals:

- Analyze concentration profiles $x(z)$ and $y(z)$
- Design packed-bed height z

Solution:

- Take a differential element of height dz differential equation
- Effective interfacial area: $A_{\text{eff}} = aSdz$
- Gas and liquid compositions both vary with z

Mass balance on control volume $z \to z + dz$

Mass balance for control volume still holds:

$$\text{In}_{liq} + \text{In}_{gas} = \text{Out}_{liq} + \text{Out}_{gas} \tag{1}$$

Governing equation for a slab with thickness dz:

$$d(Vy) = d(Lx) \tag{2}$$

Flux equation in each phase

The relation $d(Vy) = d(Lx)$ can be used to solve for each phase, if we know how to connect with **flux equations** individually.

Take the gas-phase side, the effective contact area being A_{eff} (see packed bed example in Lecture 21):

$$\begin{aligned} d(Vy_{AG}) &= d(A_{\text{eff}} N_A) \qquad (3) \\ &= \frac{k'_y}{(1-y_A)_{im}}(y_{AG} - y_{Ai})aSdz \qquad (4) \end{aligned}$$

note we need to account for the non-linear concentration profile by $y_{Bm} = (1 - y_A)_{im}$ as discussed in last lecture.

Governing differential equation for each phase

The part $d(Vy_{AG})$ can be further simplified, because we know $V = V'/(1-y_{AG})$:

$$d(Vy_{AG}) = d(V'\frac{y_{AG}}{1-y_{AG}}) \tag{5}$$

$$= V'\frac{1}{(1-y_{AG})^2}dy_{AG} \tag{6}$$

The final differential form in gas phase is then:

$$V'\frac{1}{(1-y_{AG})^2}dy_{AG} = \frac{k'_y aS}{(1-y_A)_{im}}(y_{AG}-y_{Ai})dz \tag{7}$$

Differential equation for tower height

It is often desired to know the total height Z for the tower given operating line. We can integrate over the differential equation

$$Z = \int_0^Z dz \tag{8}$$

$$= \int_{y_2}^{y_1} \frac{V'}{k'_y aS} \cdot \frac{(1-y_A)_{im}}{(1-y_{AG})^2(y_{AG}-y_{Ai})} dy_{AG} \tag{9}$$

- this is the **exact form** for the gas-side height profile
- we can obtain the same equation using liquid-side conditions
- numerical integration is needed

Different forms of height equation

If we drop the subscript A, and use $V = V'/(1-y)$ & $L = L'/(1-x)$, the height equation can be expressed in different forms

- Gas-side profile

$$Z = \int_{y_2}^{y_1} \frac{V dy}{\frac{k'_y aS}{(1-y)_{im}}(1-y)(y-y_i)}$$

- Liquid-side profile

$$Z = \int_{x_2}^{x_1} \frac{Ldx}{\frac{k'_x aS}{(1-x)_{im}}(1-x)(x_i - x)}$$

> ⚠ Warning
>
> The order of interface composition differences in liquid $(x_i - x)$ and gas $(y - y_i)$ have opposite signs!

Height equation using overall mass transfer coefficients

It is also possible to express the height equations using K'_y or K'_x:

- Gas-side profile

$$Z = \int_{y_2}^{y_1} \frac{Vdy}{\frac{K'_y aS}{(1-y)_{*m}}(1-y)(y - y^*)}$$

- Liquid-side profile

$$Z = \int_{x_2}^{x_1} \frac{Ldy}{\frac{K'_x aS}{(1-x)_{*m}}(1-x)(x^* - x)}$$

> ⚠ Warning
>
> The order of pseudo-interface composition differences in liquid $(x^* - x)$ and gas $(y - y^*)$ have opposite signs!

Simplified case: dilute system

The factors $k'_x a$, $k'_y a$, $K'_x a$, $K'_y a$ are usually not constant, making the solution harder to obtain. We will consider a simplified version of absorption of dilute gas.

- dilute regime: composition less than 0.1 (10 mol%)
- simplification 1: almost constant V and L
- simplification 2: $(1-x)_{im}/(1-x)$ and $(1-y)_{im}/(1-y)$ are almost independent on the location

- simplification 3: change of V (or L) over the column is minimal, usually use $V = (V_1 + V_2)/2$

Height equation in dilute systems

- Gas-side profile (use k'_y)

$$Z = \left[\frac{V}{k'_y aS} \frac{(1-y)_{im}}{1-y} \right] \int_{y_2}^{y_1} \frac{dy}{y - y_i}$$

- Liquid-side profile (use k'_x)

$$Z = \left[\frac{V}{k'_x aS} \frac{(1-x)_{im}}{1-x} \right] \int_{x_2}^{x_1} \frac{dx}{x_i - x}$$

Replacing $_{im}$ subscripts with $_{*m}$ and using overall coefficients gives another set of equations.

Further simplification: log mean driving force

In practical cases we can even let $\frac{(1-y)_{im}}{1-y} \approx 1$ and $\frac{(1-x)_{im}}{1-x} \approx 1$. The R.H.S. becomes only dependent on $\int_{y_2}^{y_1} \frac{dy}{y-y_i}$.

$$Z = \frac{V}{k'_y aS} \int_{y_2}^{y_1} \frac{dy}{y - y_i} \frac{V}{k'_y aS} \ln \left(\frac{y_1 - y_i}{y_2 - y_i} \right) \tag{10}$$

From the packed-bed analysis in Lecture 21 we know it corresponding to a log-mean driving force term $(y - y_i)_m$:

$$\frac{V}{S}(y_1 - y_2) = k'_y aZ \frac{(y_1 - y_i) - (y_2 - y_i)}{\ln \left(\frac{y_1 - y_i}{y_2 - y_i} \right)} \tag{11}$$

$$= k'_y aZ (y - y_i)_m \tag{12}$$

What does the result tell us?

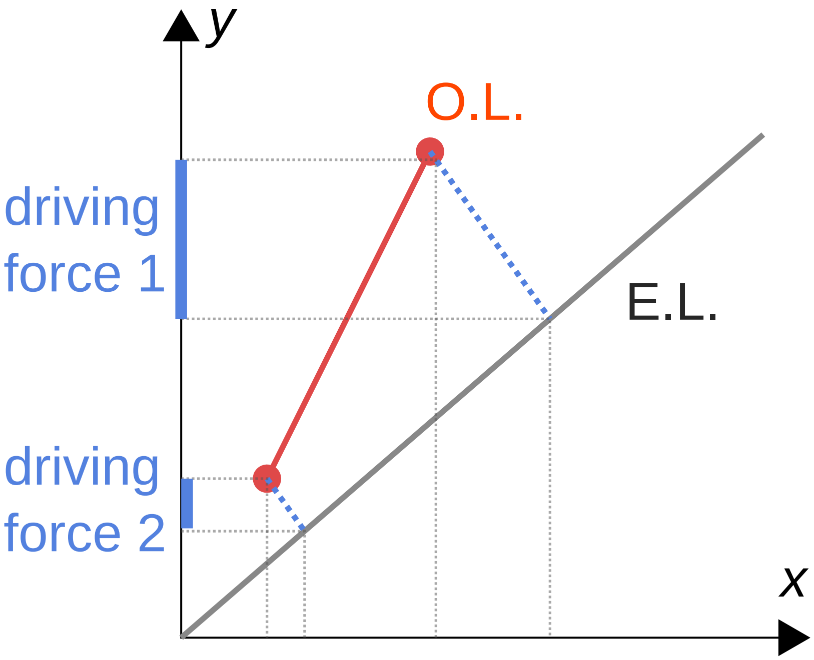
Let's take a pause and check the meaning for the governing equation

$$\frac{V}{S}(y_1 - y_2) = k'_y a Z (y - y_i)_m \tag{13}$$

- L.H.S.: molar flow of A **absorbed** per column cross-sectional area
- R.H.S.: molar flow of A **transferred** from gas to liquid phase
  - *average* driving force: log-mean of $y - y_i$
  - transfer coefficient: k'_y

Visualization of log-mean driving force

For dilute gas system, the driving force can be visualized using diagonal lines connecting the operating line and equilibrium line.



Comparison: packed bed with solid spheres

The dilute two-phase system is very close to the packed bed of solid spheres in Lecture 21, see the side-by-side comparison.

Packed bed of solid spheres

- Constant c_{Ai}
- **Uses log-mean driving force** of c

$$Q(c_2 - c_1) = k'_c a H (c_i - c)_m$$

Packed bed of liquid-gas

- Varying y_i
- **Uses log-mean driving force** of y

$$\frac{V}{S}(y_1 - y_2) = k'_y a Z (y - y_i)_m$$

All forms of height equation in dilute system

- Gas, use k'_y

$$\frac{V}{S}(y_1 - y_2) = k'_y a Z (y - y_i)_m$$

- Liquid, use k'_x

$$\frac{V}{S}(x_1 - x_2) = k'_x a Z (x_i - x)_m$$

- Gas, use K'_y

$$\frac{V}{S}(y_1 - y_2) = K'_y a Z (y - y^*)_m$$

- Liquid, use K'_x

$$\frac{V}{S}(x_1 - x_2) = K'_x a Z (x^* - x)_m$$

Example: calculate height of acetone absorption tower

(Example 10.6-4) Acetone is being absorbed by water in a packed bed column having a cross sectional area of 0.186 m^2 at 293 K and 1 atm. The inlet air contains 2.6 mol% acetone and outlet contains 0.5%. The gas flow is 13.65 kg mol inert air per hour. The pure water inlet flow is 45.36 kg mol water per hour. The coefficient $k'_y a$ is estimated to be 3.78×10^{-2} kg mol/(s $\cdot$ m^3) and $k'_x a$ is 6.16×10^{-2} kg mol/(s $\cdot$ m^3). The equilibrium line can be approximated by

$$y = 1.186x$$

a) calculate the tower height use $k'_y a$
b) repeat a) but use $k'_x a$
c) calculate the tower height use $K'_y a$

Solution steps (1)

1) Is this system dilute (<10%)?

   Yes. Choose linear operating line and height equation

2) Determine two ends of the operating line

- Point 1: x_1=?; $y_1 = 0.026$
- Point 2: $x_2 = 0$; $y_2 = 0.005$

3) Calculate x_1?

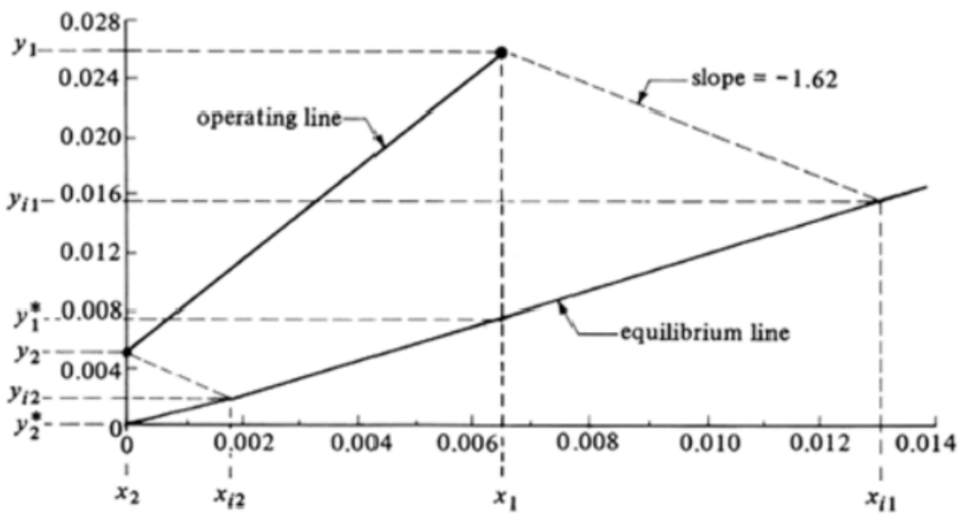
Use mass balance equation

Solution steps (2)

4) Find the slope of curve

Solution: From **Appendix A.3** for acetone–water and x_A = 0.0333 mol frac, p_A = 30/760 = 0.0395 atm or y_A = 0.0395 mol frac. Hence, the equilibrium line is $y_A = mx_A$ or 0.0395 = m (0.0333). Then, $y = 1.186x$. This equilibrium line is plotted in **Fig. 10.6-14**. The given data are L' = 45.36 kg mol/h, V' = 13.65 kg mol/h, y_1 = 0.026, y_2 = 0.005, and x_2 = 0.

Figure 10.6-14. Location of interface compositions for Example 10.6-4.



Solutions steps (3)

5) To use the height equation

$$\frac{V}{S}(y_1 - y_2) = k'_y a Z (y - y_i)_m$$

- total flow rate $V = \frac{V_1+V_2}{2} \approx 3.852 \times 10^{-3}$ kg mol/s
- log-mean driving force

$$(y - y_i)_m = \frac{(y_1 - y_i) - (y_2 - y_i)}{\ln(\frac{y_1 - y_i}{y_2 - y_i})} = 0.006$$

Final result: $Z = 1.911$ m

Summary

- Solving single-phase mass balance in absorption packed tower
- Use dilute solution for tower height analysis
- Recall the analog between the solid sphere packed bed and liquid-gas tower

$$(1-y)_{im} = \frac{(1-y_{AG})-(1-y_{Ai})}{\ln \frac{1-y_{AG}}{1-y_{Ai}}}$$

$$(y-y_i)_m = \frac{(y_1-y_{i1})-(y_2-y_{i2})}{\ln \frac{y_1-y_{i1}}{y_2-y_{i2}}}$$