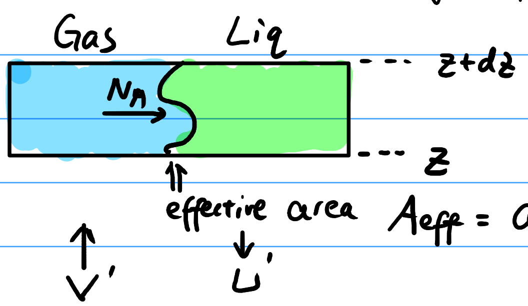
In-depth analysis for packed absorption tower!

Example: $\begin{cases} \text{Analysis of concentration profile } x(z),\ y(z) \\ \text{Design of packed bed height } z \end{cases}$



effective area $A_{eff} = a \cdot S \cdot dz$

We need to solve mass balance in 1-phase!

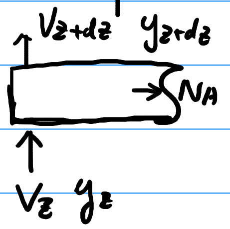
For such control volume, we get

$$In_{Liq} + In_{gas} = Out_{Liq} + Out_{gas}$$

$$\underset{\frac{kg\,mol}{s}}{d(L\cdot x)} \Leftarrow \frac{In_{liq} - Out_{liq}}{dz} = \frac{Out_{gas} - In_{gas}}{dz} \Rightarrow d(V\cdot y)$$

$\Rightarrow$ ① Gov. Eq: $\underbrace{d(V\cdot y)}_{gas} = \underbrace{d(L\cdot x)}_{Liq}$

② Each phase mass balance gives differential eq. for $d(V\cdot y)$ & $d(L\cdot x)$

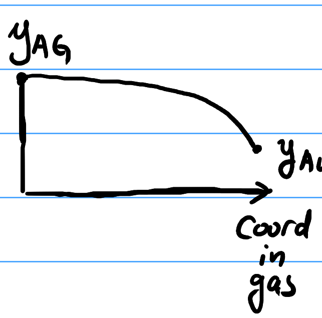


$$d(V\cdot y_{AG}) = d(A_{eff}\,N_A)$$

y_{AG} in bulk, to distinguish interface

$d(A_{eff} N_A)$?

$$= \frac{k_y'}{y_{Bm}}(y_{AG} - y_{Ai})\cdot a \cdot S \cdot dz$$



y_{Bm}: log mean of $(1-y_A)$ from bulk $y_{AG} \rightarrow y_{Ai}$

other form $(1-y_A)_{im}$ ← log mean

↑ interface

* The log-mean form is assumed for the exponential y_A profile at boundary (diffusion through non-diffusing B)

$$d(V y_{AG}) = d\left(V' \cdot \frac{y_{AG}}{1-y_{AG}}\right) = \underbrace{V'}_{\text{constant!}} d\left(\frac{y_{AG}}{1-y_{AG}}\right)$$

Combine with expression for $d(N_A \cdot A_{eff})$, we get:

$$\frac{d}{du}\left(\frac{u}{1-u}\right) = \frac{1\cdot(1-u) - u\cdot(-1)}{(1-u)^2} = \frac{1}{(1-u)^2}$$

$$V' d\left(\frac{y_{AG}}{1-y_{AG}}\right) = \frac{k_y' a}{(1-y_A)_{im}} (y_{AG} - y_{Ai}) S\, dz$$

$$V' \frac{1}{(1-y_{AG})^2} \underbrace{dy_{AG}}_{\text{variable is } y_{AG}} = \frac{k_y' a S}{(1-y_A)_{im}} (y_{AG} - y_{Ai}) dz$$

where to get y_{Ai}? from $\begin{cases} \text{eq curve!} \\ k_x'/k_y' \end{cases}$

Goal is to have tower height solved, so expression of Z

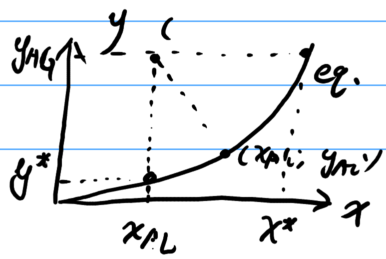
$$dz = \frac{V'}{\frac{k_y' a S}{(1-y_A)_{im}}} \cdot \frac{1}{(1-y_{AG})^2 (y_{AG} - y_{Ai})} dy_{AG}$$

$$\text{Height } Z = \int_0^Z dz = \int_{y_2}^{y_1} \frac{V'}{\frac{k_y' a S}{(1-y_A)_{im}}} \cdot \frac{1}{(1-y_{AG})^2 (y_{AG} - y_{Ai})} dy_{AG}$$

Same equation for Liquid

$$Z = \int_0^Z dz = \int_{x_2}^{x_1} \frac{L'}{\frac{k_x' a S}{(1-x_A)_{im}}} \cdot \frac{1}{(1-x_{AL})^2 (x_{AL} - x_{Ai})} dx_{AL}$$

If $\begin{cases} k_x' \to K_x' \\ k_y' \to K_y' \end{cases}$ all $x_{Ai} \Rightarrow x_A^*$, $y_{Ai} \Rightarrow y_A^*$



The previous equations are powerful, but not very easy to solve (you can use numerical integration, through!)

We need a few simplified cases

Case 1: Absorption of diluted gas mixture

Dilute ? ⇒ usually $x_A < 0.1$ (10 mol%)

What are constant? k_x' k_y', & $1 - x_A \simeq 1 - y_A \simeq 1$

Also
$$\begin{cases} V = V' \frac{1}{1-y} \simeq V' & \text{Constant} \\ L = L' \frac{1}{1-x} \simeq V' & \text{Constant} \end{cases}$$

If simplify $y = y_{AG}$ $y_i = y_{Ai}$

$$Z = \int_0^Z dz = \int_{y_2}^{y_1} \frac{V'}{\frac{k_y' a S}{(1-y_A)_{im}}} \cdot \frac{1}{(1-y_{AG})^2 (y_{AG} - y_{Ai})} d y_{AG}$$

simplify ⇓

$$Z = \left[\frac{V}{\frac{k_y' a S}{\cancel{(1-y_A)_{im}}}} \cdot \frac{1}{\cancel{(1-y)}} \right] \int_{y_2}^{y_1} \frac{dy}{y - y_i}$$

$$= \frac{V}{k_y' a S} \cdot \int_{y_2}^{y_1} \frac{dy}{y - y_i} \qquad \ln\left(\frac{y_1 - y_i}{y_2 - y_i}\right)$$

$$\frac{V}{S} = k_y' a Z \frac{1}{\ln\left(\frac{y_1 - y_i}{y_2 - y_i}\right)}$$

multiply by $(y_1 - y_2)$

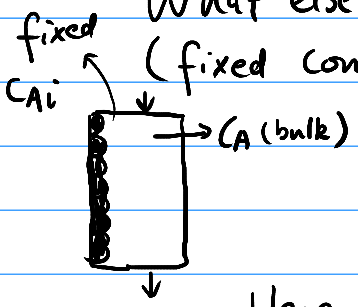
$$\frac{V}{S}(y_1 - y_2) = k_y' a Z \boxed{\frac{(y_1 - y_i) - (y_2 - y_i)}{\ln\left(\frac{y_1 - y_i}{y_2 - y_i}\right)}}$$

⇒ log mean of $(y_1 - y_i)$ $(y_2 - y_i)$

$$= k_y' a Z (y - y_i)_m$$

log mean driving force

What else system has log-mean driving force? (L21)
(fixed concentration packed bed)



$$Q(C_2 - C_1) = k_c' a H (c_i - c)_m$$

Here:

$$\frac{kgmol}{m^2 s}$$

$$\frac{V}{S}(y_1 - y_2) = k_y' a Z (y_1 - y_i)_m$$

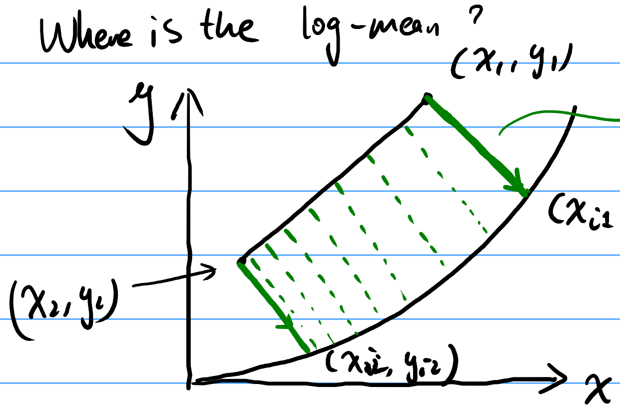
$$\frac{kgmol}{m^2 s} \cdot \frac{m^2}{m^3} \cdot m$$

varying y_i interface!

So, for dilute system with const k_y
1-phase mass balance

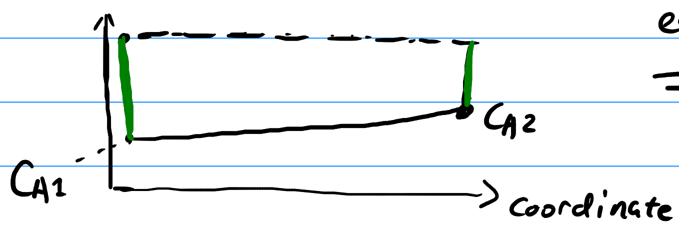
[Flow rate][Conc diff] = [Mass Transfer coeff]·[Aeff]·[log mean D.F.]

Where is the log-mean?



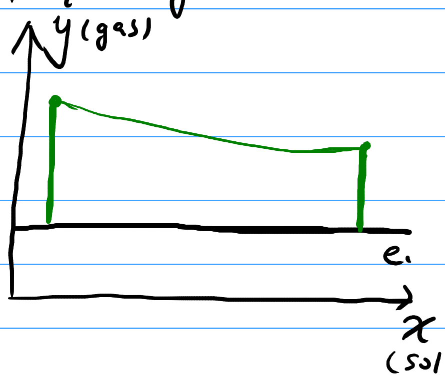
$(1 - x \simeq 1 - y \simeq 1)$

Compare in fixed conc packed bed



equivalent eq. diagram

$\Longrightarrow$



Other forms

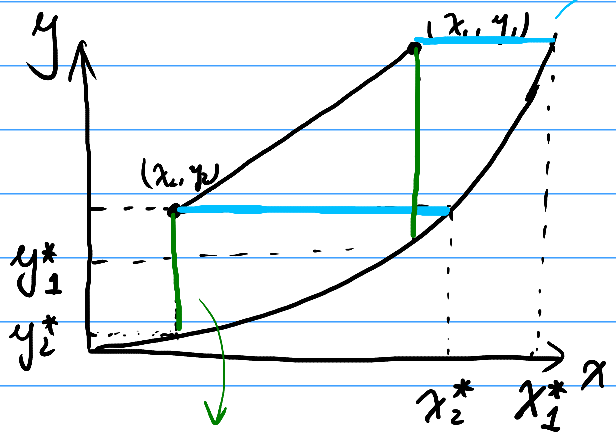
liquid: $$\frac{L}{S}(x_1 - x_2) = k_x' a Z (x_i - x)_m$$

Using overall coefficients

Gas $$\frac{V}{S}(y_1 - y_2) = K_y' a Z (y - y^*)_m$$

Liq $$\frac{L}{S}(x_1 - x_2) = K_x' a Z (x^* - x)_m$$

Liquid driving force log mean $(x^* - x)$



gas driving force log mean $(y - y^*)$

How do we see V & L?

Usually $$\begin{cases} V = \dfrac{V_1 + V_2}{2} \\ L = \dfrac{L_1 + L_2}{2} \end{cases}$$