CHE 318 Lecture 26

Designing Packed Beds For Concentrated Systems

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2026-03-13

Learning outcomes

After this lecture, you will be able to:

- **Recall** the general procedure for designing a packed tower in concentrated systems.
- **Describe** the concepts of transfer unit and number of transfer units in packed-bed design.
- **Apply** realistic concentration profiles to estimate tower height.

Cheatsheet for packed bed design

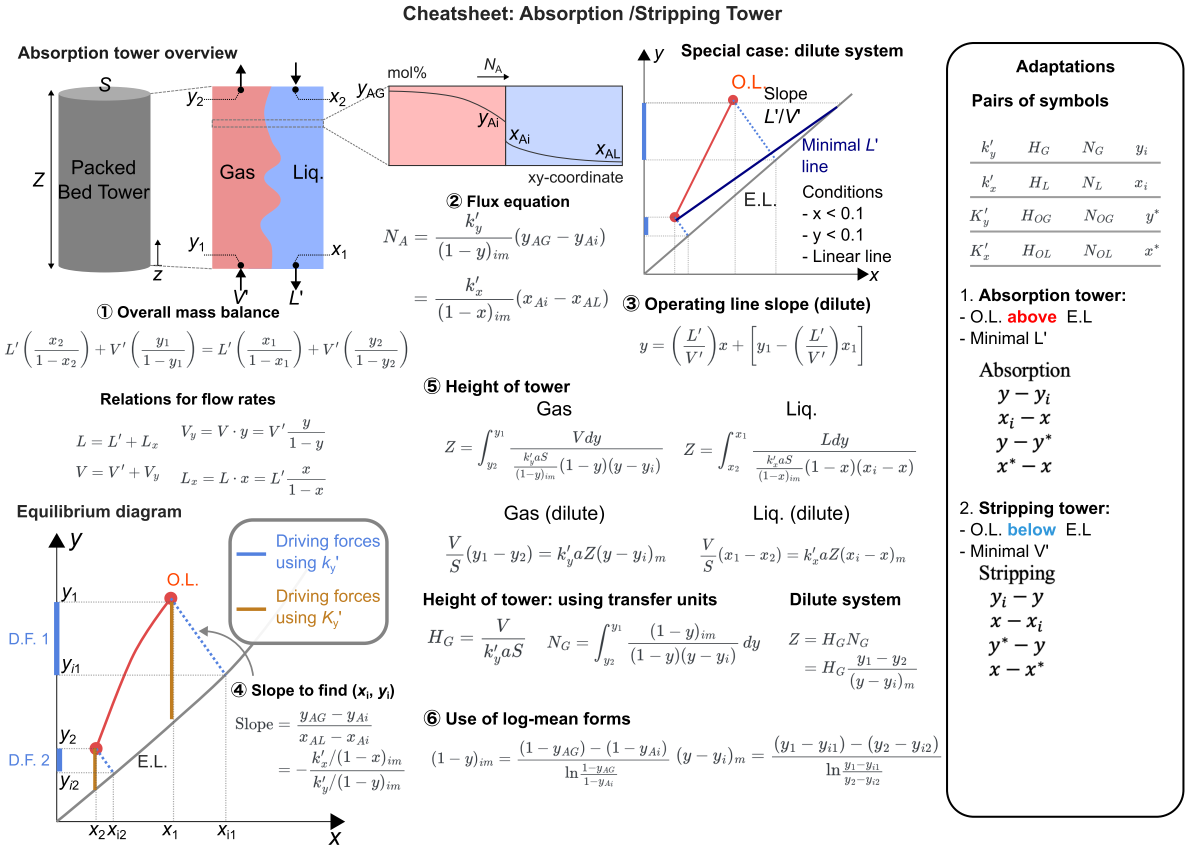


Figure 1: Physical copies distributed in class

Recall: packed tower height equation (individual phase)

General equation for gas. What the meanings for each term?

$$Z = \int_{y_2}^{y_1} \frac{V'}{k'_y a S} \cdot \frac{(1-y)_{im}}{(1-y)^2(y-y_i)}\, dy \tag{1}$$

$$= \int_{y_2}^{y_1} \frac{V}{k'_y a S} \cdot \frac{(1-y)_{im}}{(1-y)(y-y_i)}\, dy \tag{2}$$

Recall: height equation for diluted system

If $x < 0.1$ and $y < 0.1$, typically use (for gas phase):

$$Z = \left[\frac{V}{k'_y a S} \frac{(1-y)_{im}}{1-y} \right] \int_{y_2}^{y_1} \frac{dy}{y - y_i}$$

or even simpler

$$Z = \left[\frac{V}{k'_y a S} \right] \int_{y_2}^{y_1} \frac{dy}{y - y_i}$$

what insights can we get?

Motivation: transfer units for packed beds

From unit analysis, the term $\frac{V}{k'_y a S}$ characterizes the height. We often use the concept of **transfer unit** to determine the performance of mass transfer in each phase. For gas phase, we have the "height" of the transfer unit H_G as

$$H_G = \frac{V}{k'_y a S} \tag{3}$$

- H_G is controlled by ratio of flow rate and mass transfer in gas phase
- smaller H_G means more efficient packing (thus needing shorter tower)

Define the number of transfer units

The rest of the equation for Z can be rewritten as "number of transfer units", similar to the theoretical number of trays in a tray tower. For gas phase general case, we have

$$N_G = \int_{y_2}^{y_1} \frac{(1-y)_{im}}{(1-y)(y-y_i)} \, dy \tag{4}$$

For dilute system, we can approximate N_G by

$$N_G = \int_{y_2}^{y_1} \frac{dy}{y - y_i} \tag{5}$$

$$= \frac{y_1 - y_2}{(y - y_i)_m} \tag{6}$$

Transfer unit view of packed bed

For dilute system, we can see that

$$Z = H_G N_G \tag{7}$$

$$= H_G \frac{y_1 - y_2}{(y - y_i)_m} \tag{8}$$

- Total height Z is a stack of "transfer units" with height H_G
- Number of the units is governed by the total absorption $(y_1 - y_2)$ and driving force $(y - y_i)_m$
- Need to absorb more (larger $(y_1 - y_2)$)? higher tower
- Operation line close to equilibrium? higher tower

Example 1: packed bed height estimation using transfer units

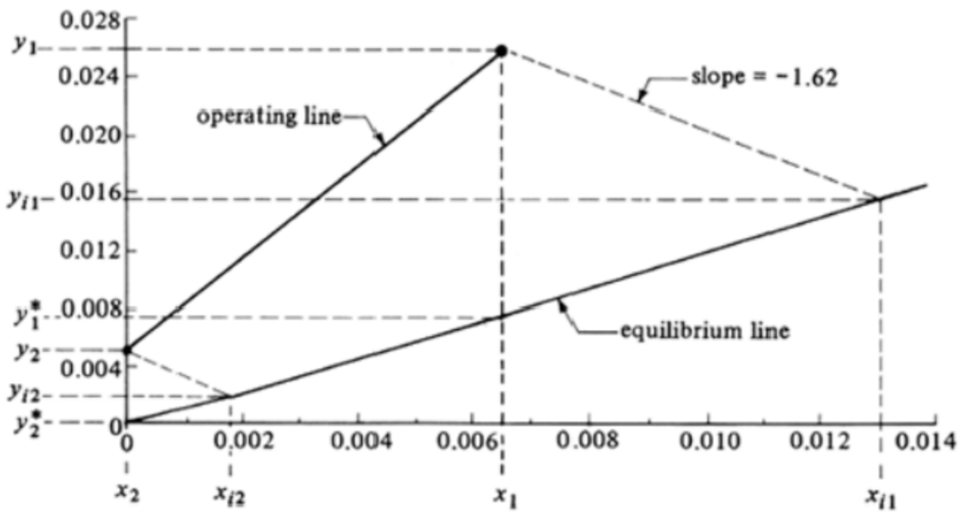
Similar example in Lecture 24, Acetone is being absorbed by water in a packed bed column having a cross sectional area of 0.186 m^2 at 293 K and 1 atm. The inlet air contains 2.6 mol% acetone and outlet contains 0.5%. The gas flow is 13.65 kg mol inert air per hour. The pure water inlet flow is 45.36 kg mol water per hour. The coefficient $k'_y a$ is estimated to be 3.78×10^{-2} kg mol/$(\text{s} \cdot \text{m}^3)$ and $k'_x a$ is 6.16×10^{-2} kg mol/$(\text{s} \cdot \text{m}^3)$. The equilibrium line can be approximated by $y = 1.186x$.

a) Now calculate the height using the dilute regime equation
b) Now calculate the height using H_G and N_G statements

Example 1 diagram

Solution: From **Appendix A.3** for acetone–water and x_A = 0.0333 mol frac, p_A = 30/760 = 0.0395 atm or y_A = 0.0395 mol frac. Hence, the equilibrium line is $y_A = mx_A$ or 0.0395 = m (0.0333). Then, $y = 1.186x$. This equilibrium line is plotted in **Fig. 10.6-14**. The given data are L' = 45.36 kg mol/h, V' = 13.65 kg mol/h, y_1 = 0.026, y_2 = 0.005, and x_2 = 0.

Figure 10.6-14. Location of interface compositions for Example 10.6-4.



Example 1 solution

Key numbers during calculation:

1. Liquid outlet $x_1 = 0.00648$
2. $(1-y)_{im} \approx (1-x)_{im} \approx 1$ use dilute regime is justified
3. $(y - y_i)_m = 0.00602$
4. Average $V = (V_1 + V_2)/2 = 3.852 \times 10^{-3}$ kg mol/s

Final results:

1. $Z = 1.911$ m
2. $H_G = 0.548$ m, $N_G = 3.488$

Practical estimation of k_y'a or H_G

In real packed beds, coefficients are often obtained from correlations (using dimensionless numbers etc), which may depend on:

- N_{Re}, N_{Sc}
- Weight flow rates G_y, G_x
- Packing type (shape? diameter?)

Design of packed bed in realistic situations

In many industrial applications, engineers are interested in one of the approaches to estimate the desired tower height:

1. Estimate the correlation of $k'_y a$ and $k'_x a$ as functions of G_x and G_y

$$k'_y a = f_y(G_x, G_y, \text{pack type}); \quad k'_x a = f_x(G_x, G_y, \text{pack type})$$

2. Estimate H_G (and / or H_L) from the correlation

$$H_G = f_G(G_x, G_y, N_{Sc}, \text{pack type})$$

Either way, the flow rates are locally dependent, so integration should be used. Let's see a detailed example from the textbook.

Example 2: concentrated packed bed design

(Example 10.7-1) A tower packed with 25.3 mm ceramic rings is used to absorb SO_2 from air using pure water at 293 K and 1 atm. The entering gas contains 20 mol% SO_2 and leaving gas has 2 mol%. The inert air flow is 6.53×10^{-4} kg mol/s and inlet water flow is 4.20×10^{-2} kg mol/s. Cross sectional area of the tower is 0.0929 m^2. From literature, the film mass transfer coefficients are estimated as

$$k'_y a = 0.0594 G_y^{0.7} G_x^{0.25} \qquad k'_x a = 0.152 G_x^{0.82}$$

where G_x and G_y are total weight of liquid / gas flow per second per area, respectively. Estimate the tower height needed.

Example 2: steps?

We have seen $y_2 = 0.2$, so in general the dilute solution is not accurate. Use the following steps instead:

1. **At each point** on the operating line: determine (x, y)
2. Use local flow rates V_y, L_x to find
   - $G_y = (M_{\text{air}} V' + M_A V_y)/S$
   - $G_x = (M_{\text{water}} L' + M_A V_x)/S$
3. Calculate $k'_y a$ and $k'_x a$ for each point
4. Use trial-and-error / numerical method to determine (x_i, y_i)
5. Integrate through the bed to get Z

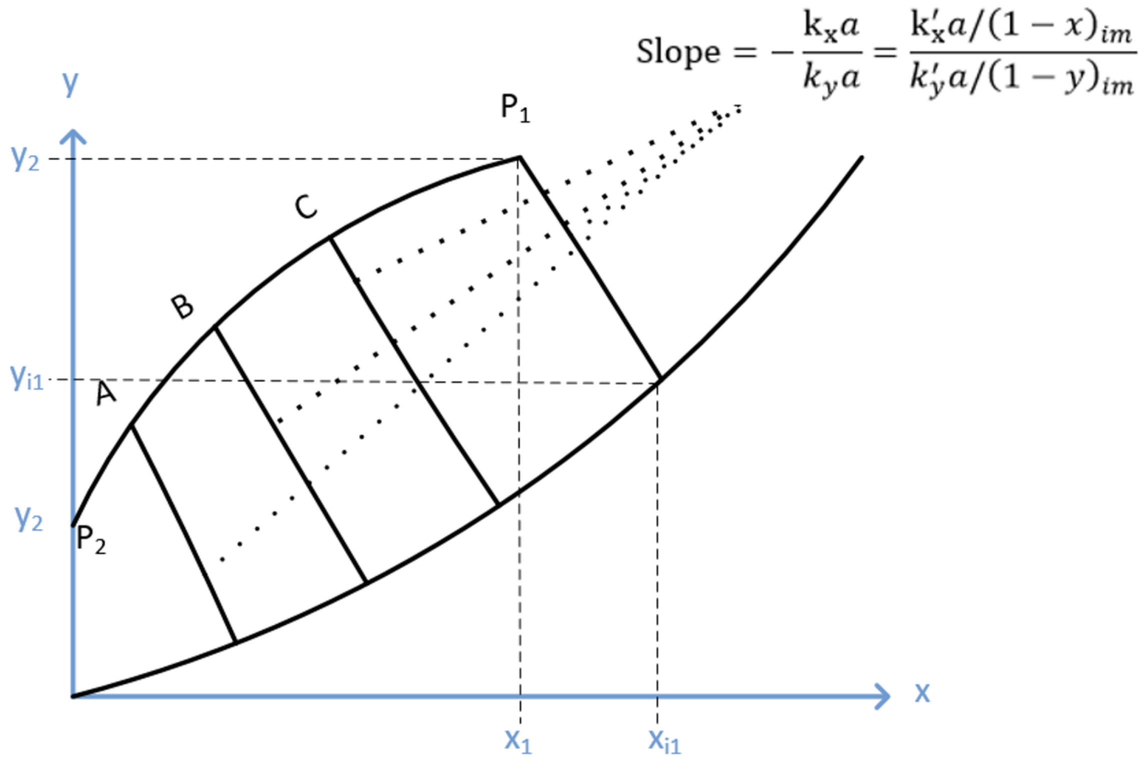
Notes on the integration

The height equation is basically integration of a function $f(y)$ over the column. Manual integration using trapezoidal rule usually require at least 5 points along y

$$Z = \int_{y_2}^{y_1} \frac{V dy}{\frac{k'_y a S}{(1-y)_{im}}(1-y)(y-y_i)} \tag{9}$$

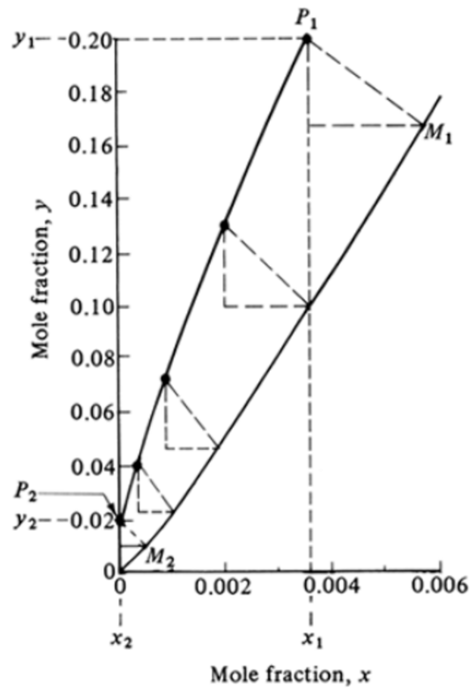
$$= \int_{y_2}^{y_1} f(y) dy \tag{10}$$

$$= \sum_{j=0}^{n-1} [f(y_j) + f(y_{j+1})] \frac{y_{j+1} - y_j}{2} \tag{11}$$


$\text{Slope} = -\frac{k_x a}{k_y a} = \frac{k'_x a/(1-x)_{im}}{k'_y a/(1-y)_{im}}$
y
P_1
y_2
C
B
y_{i1}
A
y_2
P_2
x
x_1
x_{i1}

Example 2: operating line

Figure 10.7-1. Operating line and interface compositions for Example 10.7-1.



Example 2: textbook solutions (1)

Solution: The given data are $V' = 6.53 \times 10^{-4}$ kg mol air/s (5.18 lb mol/h), $L' = 4.20 \times 10^{-2}$ kg mol/s (333 lb mol/h), $y_1 = 0.20$, $y_2 = 0.02$, and $x_2 = 0$.

Substituting into the overall material-balance **equation (10.6-4)**,

$$L'\left(\frac{x_2}{1 - x_2}\right) + V'\left(\frac{y_1}{1 - y_1}\right) = L'\left(\frac{x_1}{1 - x_1}\right) + V'\left(\frac{y_2}{1 - y_2}\right)$$

$$4.20 \times 10^{-2}\left(\frac{0}{1 - 0}\right) + 6.53 \times 10^{-4}\left(\frac{0.2}{1 - 0.2}\right) = 4.20 \times 10^{-2}\left(\frac{x_1}{1 - x_1}\right) + 6.53 \times 10^{-4}\left(\frac{0.02}{1 - 0.02}\right)$$

Solving, $x_1 = 0.00355$. The operating line **Eq. (10.6-5)** is

$$4.20 \times 10^{-2}\left(\frac{x}{1 - x}\right) + 6.53 \times 10^{-4}\left(\frac{0.2}{1 - 0.2}\right) = 4.20 \times 10^{-2}\left(\frac{0.00355}{1 - 0.00355}\right) + 6.53 \times 10^{-4}\left(\frac{y}{1 - y}\right)$$

Setting $y = 0.04$ in the operating-line equation above and solving for x, $x = 0.000332$. Selecting other values of y and solving for x, points on the operating line were calculated as shown in **Table 10.7-1** and plotted in **Fig. 10.7-1** together with the equilibrium data from **Appendix A.3**.

Example 2: textbook solutions (2)

Table 10.7-1. Calculated Data for Example 10.7-1

y	x	V	L	G_y	G_x	k_y'a	k_x'a	x_i	y_i	1 − y	$(1 - y)_{iM}$	$y - y_i$	f(y)
0.02	0	6.66×10^{-4}	0.042	0.213	8.138	0.03398	0.848	0.00046	0.009	0.98	0.985	0.011	19.25
0.04	0.00033	6.80×10^{-4}	0.04201	0.2226	8.147	0.03504	0.849	0.00103	0.0235	0.96	0.968	0.0165	12.77
0.07	0.00086	7.02×10^{-4}	0.04203	0.2378	8.162	0.03673	0.85	0.00185	0.0476	0.93	0.941	0.0224	9.29
0.13	0.00201	7.51×10^{-4}	0.04208	0.2712	8.196	0.04032	0.853	0.00355	0.1015	0.87	0.885	0.0285	7.16
0.2	0.00355	8.16×10^{-4}	0.04215	0.3164	8.241	0.04496	0.857	0.00565	0.1685	0.8	0.816	0.0315	6.33

Where,

$$f(y) = \frac{V \cdot dy}{\frac{k'_y a S}{(1-y)_{im}} \cdot (1-y) \cdot (y - y_i)}$$

Final integration result: $Z = 1.588$ m

Summary

In this lecture we have seen how the height of a realistic packed bed tower is calculated and how to obtain relative statements using the transfer unit concepts.

In next weeks, we will talk about the final topic: mass transfer coupled with heat transfer. One useful example will be the humidification tower.