Question: humidifier design

A living room of $60\ m^2$ at $22°C$ has eq. humidity of 20%, how much water is required to humidify the whole room to a final humidity of 45%? Assume height of ceiling is 3m.

Humidity sensor $\Rightarrow$ $H_R = \dfrac{P}{P_{vap}}$

$\Delta p = (H_{R1} - H_{R0}) \cdot P_{vap}$ $\qquad$ $PV = nRT = \dfrac{m_A}{M_A} RT$

$$\Delta m_A = \frac{\Delta P V M_A}{RT} = \frac{(H_{R1} - H_{R0}) \cdot P_{vap} \cdot V \cdot M_A}{RT}$$

P_{vap} $22°C$ $= 2.64\ kPa$

$$= \frac{(0.45 - 0.20) \cdot 2.64 \times 10^3 \times (60 \times 3) \times 18.02}{8314 \times (273.15 + 22)}$$

$$= 0.87\ kg \quad (870\ mL)$$

This is a lower-bound estimation. In reality you need humidifier to have water tank >> 870 mL

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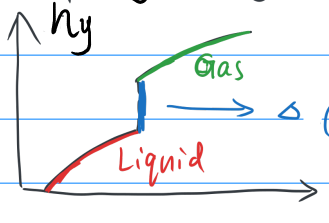
⑦ Enthalpy (of mixed water vapour - air)

Enthaphy h_y: definition $\dfrac{\partial h_y}{\partial T} = C_s$ (specific heat) (text book uses H_y, but I'd like to distinguish between enthalpy & humidity)

Unit of h_y? Joule / Calories



→ Δ (Gas − Liquid) ⇒ latent heat

Latent heat is a measurement of entropy difference (a big topic)

For out application

$$y = \underbrace{C_s}_{\text{Specific heat}} \underbrace{(T - T_0)}_{\text{Temp diff}} + \underbrace{H}_{\text{abs. humidity}} \underbrace{\lambda_0}_{\text{latent heat}}$$

You can see in continuous range $\left(\frac{\partial h_y}{\partial T} = C_s\right)$

latent heat is associated with if increase humidity

H ↑ h_y ↑

⇗ T uses Celsius

Air/H_2O mixture: $h_y = (1.005 + 1.88\,H)\,T + 2501.4\,H$ at 0°C

$\frac{\partial h_y}{\partial H} = 1.88T + 2501.4$ when temp ↑ more heat is required to humidify

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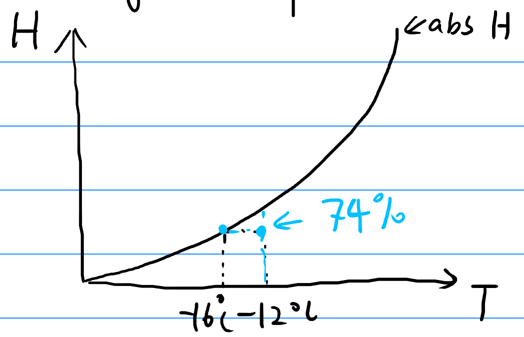
Myth busting time!

1. My weather app shows outside is −12°C & 74% humidity, dew point −16°C. what does that mean?



2. In this scenario (−12°C, 74%), cold air comes in my house and humidity inside house immediated drops. why?

$P_{vap}(T=-12°C) = 0.24$ kPa

$P_{vap}(T=22°C) = 2.64$ kPa

10 times difference!

74% RH at −12°C = $p = 0.24 \times 0.74 = 0.178$ kPa

$$\boxed{H_R(22°C) = \frac{0.178}{2.64} = 6.7\% \,!}$$

In other words, { cold air → appear dry in high T
hot air → appear humid in low T

3. Why does "humid hot"" feel worse than "dry hot"?
(E.g. compare 35°C in rainforest vs 35°C in desere)
R.H = 95% R.H. = 25%

Link to evaporation of human body. Can evaporation be effective in this scenario?

$P_{vap} \simeq 5.6$ kPa $H = \frac{18.02}{28.97} \frac{P_A}{P_T - P_A}$

$R.H. = 95\% \Rightarrow H = 0.0345$
$R.H. = 25\% \Rightarrow H = 0.00872$

123.74 kJ/kg
$h_y(95\%) = 37.45 + 87.30$
$h_y(25\%) = 57.56$ kJ/kg
$35.75 + 21.81$
$\Uparrow$ gas specific heat $\Uparrow$ latent heat

① humid air's enthalpy (thermal energy) is much more than dry air
② when $T_{air} \simeq T_{skin}$ only driving force is R.H. difference.
N_A from skin is smaller is hot & humid air!

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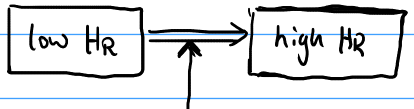
Humidification process: dynamic evaporation

From last lecture we know that vapour press of H_2O varies by T ⇒ relative humidity changes a lot!

The reversal process: evaporation takes away heat can also be explained by the enthalpy concept.



increase enthalpy / additional heat moves into air

because the heat is associated with water, water temperature will drop

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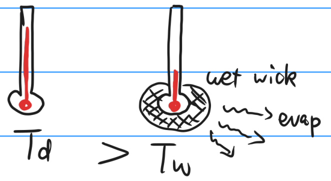
- Final concept we'll introduce is the "wet-bulb temp" T_w

In midterm we talk about interfacial temp ↓ due to evaporation

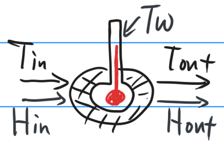
at steady state, when the interface in dry air is T_d, given the S.S. humidity, we will measure a decreased Temp T_w

$T_w < T_d$



(Conceptual experiment T_w is universal lower bound for humidification process).

Wet bulb setup analysis



At steady state (a big room) with convection

① $T_{in} = T_{out}$, but $T_w < T_{in}$

② Presumably $H_{out} > H_{in}$

③ What's happening? $\underset{\substack{\uparrow \\ \text{enthalpy}}}{H_y} = C_s(T - T_0) + H\lambda_0$

if H humidity changes, the latent heat flows from the thermometer → air flow but not essentially change T

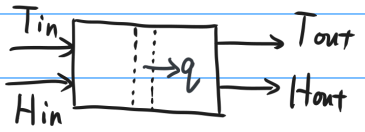
Useful concepts from last lecture

$$h_y = (1.005 + 1.88\,H)\,(T - T_0) + 2501.4\,H$$

⇑ enthalpy 1 kg dry air

↑ abs humidity measured in $\frac{\text{kg } H_2O}{\text{kg air}}$

Wet-bulb experiment: flows (needs some basic heat transfer)



For this control volume, write a heat balance eqn.

Heat flux $\vec{q}$ from L → R, which balances

$$Q_{in} - Q_{out} = \text{Latent}_{in} - \text{Latent}_{out}$$

$$q = m_A \cdot \lambda_w \cdot A \cdot N_A$$

- m_A mol. weight of H_2O (18 kg/kgmol)
- λ_w latent heat of evaporation @ T_w (kJ/kg H_2O)
- N_A total mass flux of H_2O kg mol/s

How do we get N_A? (Mass Transfer coeff)

$$N_A = \boxed{\frac{k_y'}{y_{Bm}}}(y_w - y)$$

⇒ $k_y'/y_{Bm} = k_y$ $\quad y_{Bm} \simeq 1$

y_w: mol. frac at bulb interface (T_w)
y: molar fraction in bulk gas

For humidity, need to convert y from H

$$y = \frac{[\text{mol of } H_2O]}{[\text{mol of } H_2O] + [\text{mol. of air}]} = \frac{H/m_A}{1/m_B + H/m_A}$$

$$\simeq \frac{m_B}{m_A} H$$

(H not R.H. so usually small)

Rewrite $q = m_A \lambda_w A N_A$

L.H.S. $\doteq \cancel{m_A} \cdot \lambda_w \cdot A \cdot \frac{m_B}{\cancel{m_A}} \cdot \underline{\underline{k_y}} \cdot (H_w - H)$

effective k in stagnant B

Fourier's law

$q = \underline{\underline{h}}(T - T_w) \cdot A$

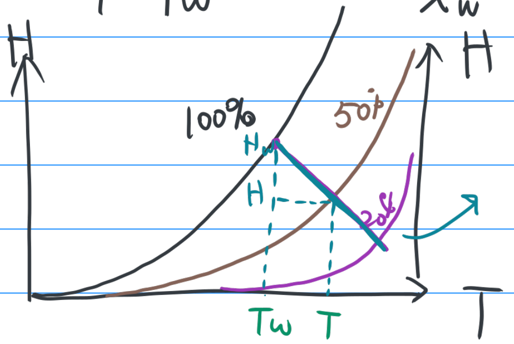
heat transfer coeff

Rearrange everything

heat transfer

mass transfer

$$\frac{H - H_w}{T - T_w} = -\frac{h / m_B k_y}{\lambda_w}$$



Slope $-\frac{\frac{h}{m_B k_y}}{\lambda_w}$

Hp not HR

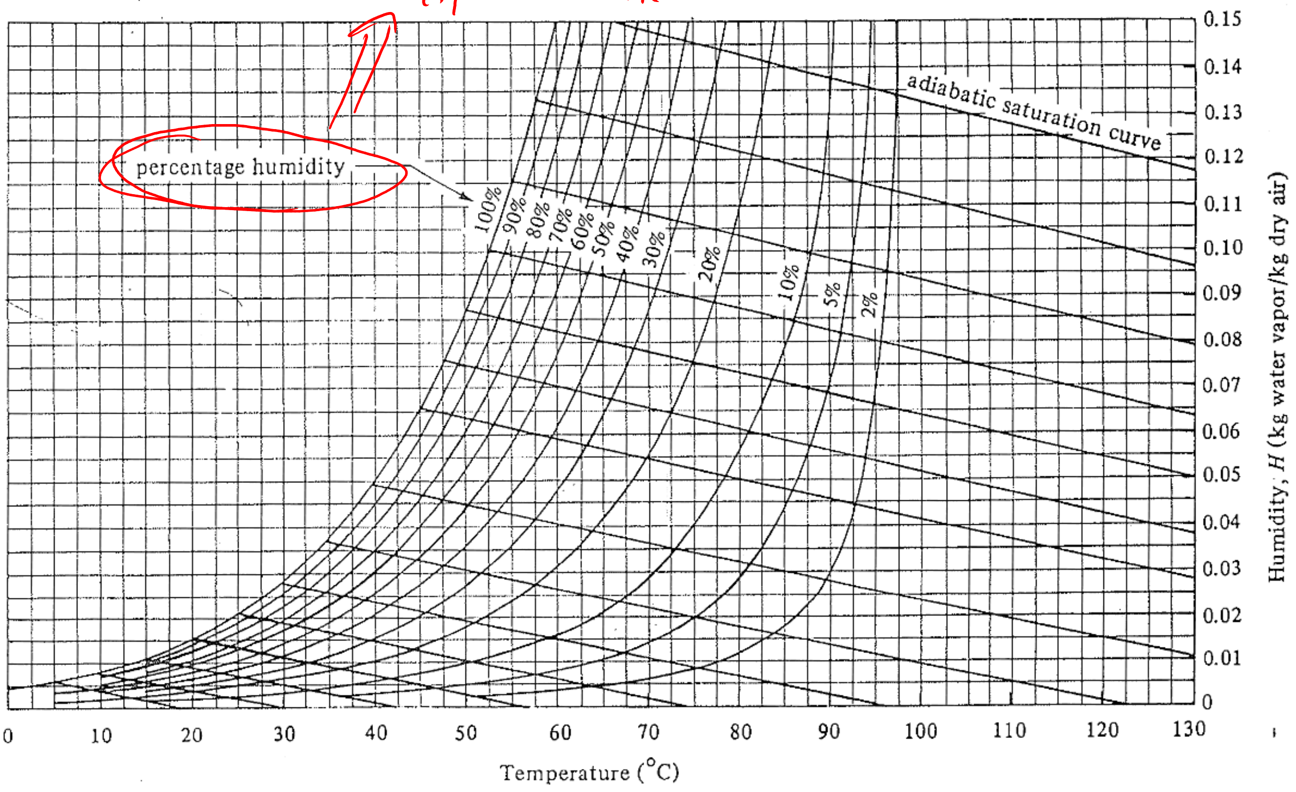


FIGURE 9.3-2. *Humidity chart for mixtures of air and water vapor at a total pressure of 101.325 kPa (760 mm Hg). (From R. E. Treybal, Mass-Transfer Operations, 3rd ed. New York: McGraw-Hill Book Company, 1980. With permission.)*

Look at the equation again → heat transfer coefficient

$$\frac{H-H_w}{T-T_w} = -\frac{h/(M_B k_y)}{\lambda_w}$$

↑ w: wet-bulb

Ratio $\frac{h}{M_B k_y}$ = psychrometric ratio (psychro- = cold, metric = measure) measure wet bulb

In the adiabatic (no external heat exchange) process

Heat balance:

$$H, T \longrightarrow \boxed{\text{water}} \longrightarrow H_s\ T_s$$

saturated temperature T_s

Coincidentally, $\left\{\begin{array}{l} \frac{h}{M_B k_y} \simeq 0.96 - 1.005 \\ C_s \simeq 1.005 + 1.88 \cdot H \end{array}\right.$

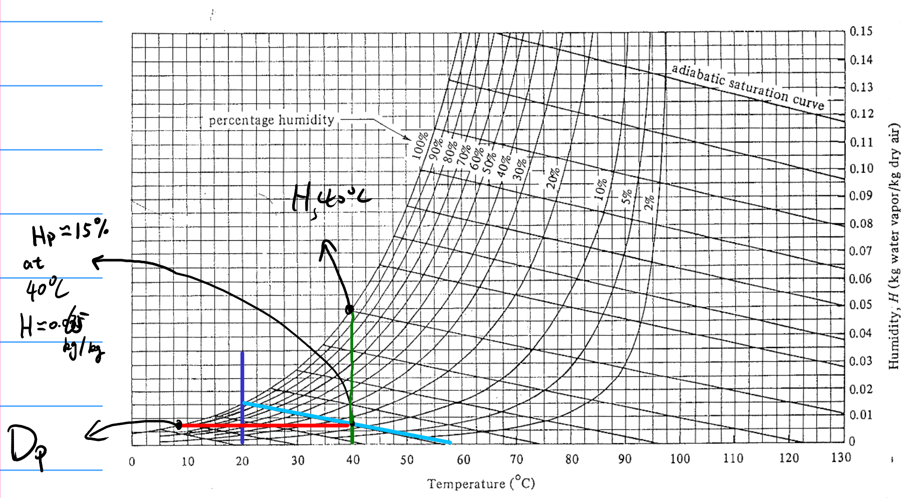
$\boxed{\text{use the "adiabatic line"}}$ ↓ T_w

Let's use the humidity chart for some analysis

Humid air at 40°C has a wet bulb temp of 20°C

what is H in current air, and calulate H_P, Dew point, V_H

humid heat & enthalpy



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$H(20°C, sat) = 0.0147$ kg w/kg air

Follow the chart

$$\frac{H-H_w}{T-T_w} = -\frac{1.005+1.88H}{\lambda_s}$$

or to solve

$$C_s(T-T_s) + H\lambda_s = C_s(T_s-T_s) + H_s\lambda_s$$

$$\frac{H-H_w}{T-T_w} = -\frac{1.005+1.88H}{2501.4}$$

$H(40°C) = 0.0065$

Percentage humidity

at 40℃ $H_{(40℃)} = 0.049$

$H_p = \frac{0.0065}{0.049} = 13\%$

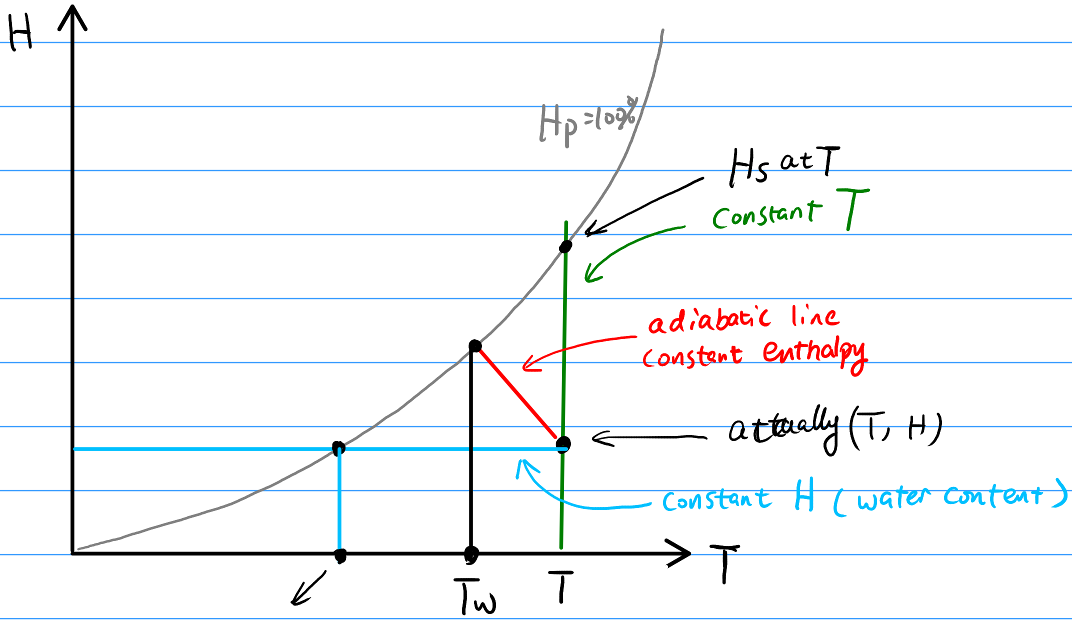
Relative humidity

$H_R = \frac{P_A}{P_T - P_A} \simeq 14\%$

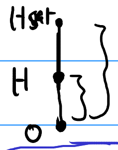
Dew point : around 8℃

$V_H \doteq 0.90$ m³/kg dry air (use H at 40℃)

$C_s = 1.005 + 1.88 H \doteq 1.02$ kJ /(dry air) /



Hp:



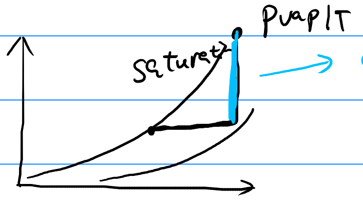
$H_p = H_R \cdot \frac{P_T - P_{vap}}{P_T - P_A}$

Hp slight smaller than HR

What is the indication of the wet bulb temperature ?

① Cooling of water using dry air is possible (max cooling Tw)

② vapour pressure difference is driving force (see previous discussion of humid hot)



→ driving force

this is the concept behind cooling tower