Look at the equation again → heat transfer coefficient

$$\frac{H-H_w}{T-T_w} = -\frac{h/(M_B k_y)}{\lambda_w}$$

↑ w: wet-bulb

Ratio $\frac{h}{M_B k_y}$ = psychrometric ratio (psychro- = cold ; metric = measure) measure wet bulb

In the adiabatic (no external heat exchange) process

Heat balance:

$H, T \rightarrow$ [water] $\rightarrow H_s\ T_s$

saturated temperature T_s

Coincidentally, $\begin{cases} \frac{h}{M_B k_y} \simeq 0.96 - 1.005 \\ C_s \simeq 1.005 + 1.88 \cdot H \end{cases}$

use the "adiabatic line" ↓ T_w

Let's use the humidity chart for some analysis

Humid air at 40°C has a wet bulb temp of 20°C

what is H in current air, and calulate H_p, Dew point, V_H

humid heat & enthalpy

$H(20°C, sat) = 0.0147$ kg w/kg air

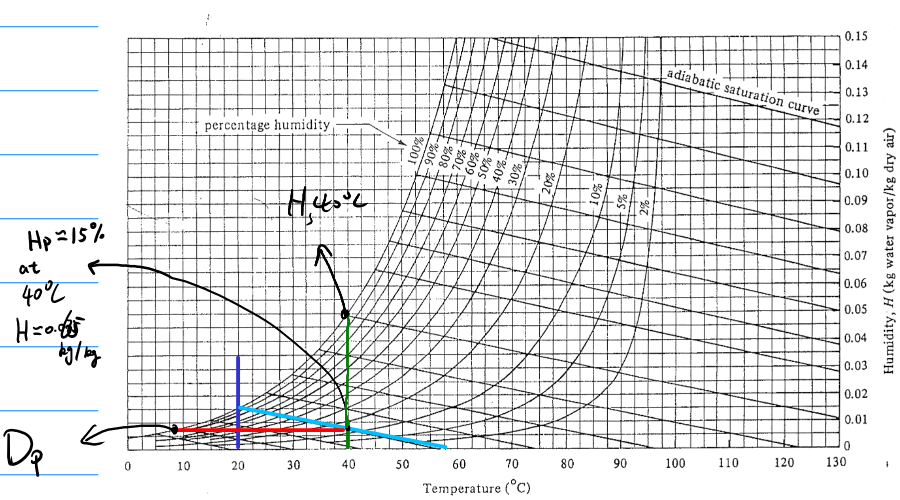


FIGURE 9.3-2. Humidity chart for mixtures of air and water vapor at a total pressure of 101.325 kPa (760 mm Hg). (From R. E. Treybal, Mass-Transfer Operations, 3rd ed. New York: McGraw-Hill Book Company, 1980. With permission.)

Follow the chart

$$\frac{H-H_w}{T-T_w} = -\frac{1.005+1.88H}{\lambda_s}$$

or to solve

$$C_s(T-T_s) + H\lambda_s = C_s(T_s - T_s) + H_s\lambda_s$$

$H(40°C) = 0.0065$

$$\frac{H-H_w}{T-T_w} = -\frac{1.005+1.88H}{2501.4}$$

Percentage humidity

at 40°C $\quad H_{(40°C)} = 0.049$

$$H_p = \frac{0.0065}{0.049} = 13\%$$

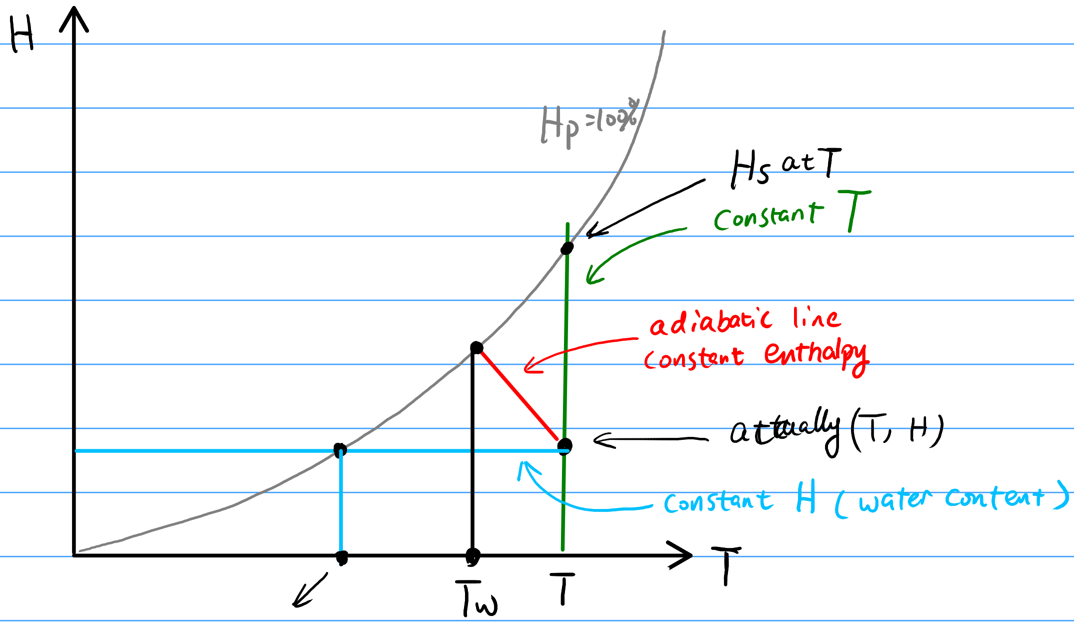
Relative humidity

$$H_R = \frac{P_A}{P_T - P_A} \approx 14\%$$

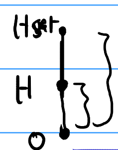
Dew point : around 8°C

$v_H \doteq 0.90\ m^3/kg$ dry air $\quad$ (use H at 40°C)

$C_s = 1.005 + 1.88H \doteq 1.02\ kJ/(dry\ air)/$



Hp:

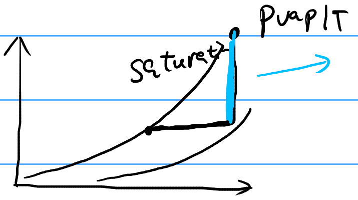


$$H_p = H_R \cdot \frac{P_T - P_{vap}}{P_T - P_A}$$

Hp slight smaller than HR

What is the indication of the wet bulb temperature ?

1. Cooling of water using dry air is possible (max cooling T_w)
2. vapour pressure difference is driving force (see previous discussion of humid hot)



→ driving force

this is the concept behind cooling tower

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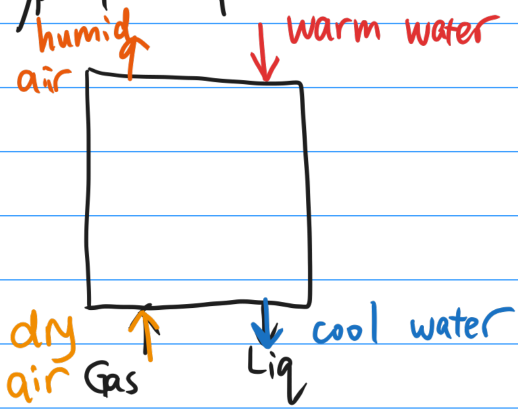
Cooling tower introduction

ooling tower takes the principle of humidification process

Put more water into air ⟶ requires heat

↑ absorbes heat from liquid

Typical setup



why cooling ?

① dry air → evaporation (below Hvap)

② latent heat flow: from water → air

① Minimal temp during cooling ? T_w

② Why evaporation occurs ?

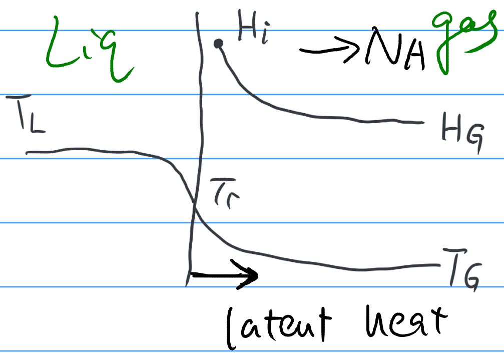
$$P_{vap}\big|_{T} - P_A\big|_{T_w} > 0$$

③ usual setup: get water flow rate ↳ constant air flow can be forced (like blowing a cup of hot water)

④ Sensitive to air-water contact

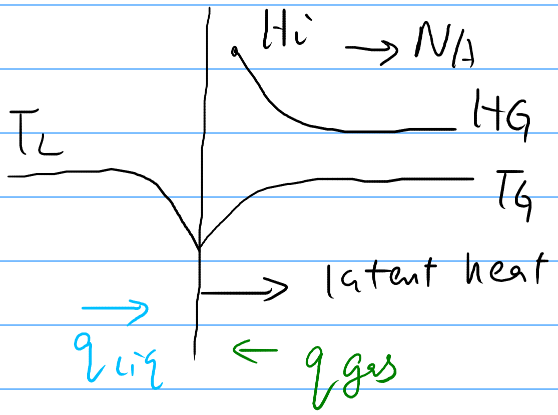
Design analysis for cooling tower

Because both heat / mass transfer occur, draw diagram

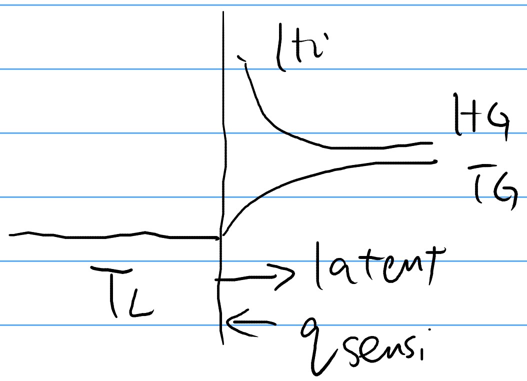


Top of column

(T_G is low)



middle of column



mass transfer always liq → gas

but T profile different

How to solve?

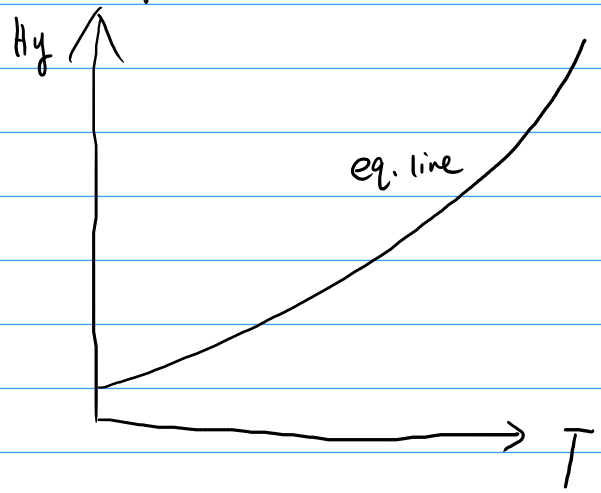
want to know

① Given liq flow rate, gas flow rate ⇒ final T

③ Design tower height

We have similar operating line in cooling towers

Instead of using humidity, use H_y (enthalpy of mixture)



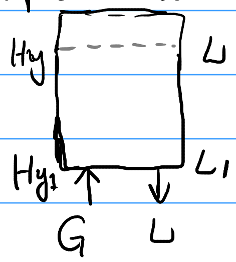
$\leftarrow H_y = (1.005 + 1.88\, H_s) + H_s \cdot 2501.4$ $\underline{\underline{kJ/kg}}$

H_s at each T from psychrometric chart

Careful unit!

If currently not saturated $H < H_s$, point below eq. line

Step 2: what is operating line?



Control volume from bottom to location &

(H_{y1}, G, L_1) (H_y, L)

Since only very small amount is evaporated

$L_1 \simeq L = \text{const}$

Heat balance

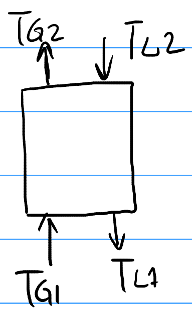
Heat in = Heat out

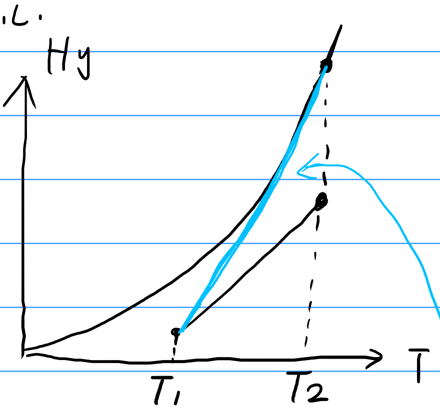
$$G(\underbrace{H_y - H_{y1}}_{\text{Diff in Enthalpy}}) = L \underbrace{C_L}_{\text{heat cap. in liquid}} (T_L - T_{L1})$$

describes the O.L.

$C_L = 4.187$ kJ/kg·K

$\doteq 1$ kcal/kg·K





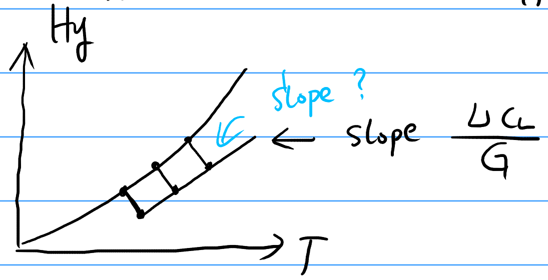
Slope? $\boxed{\dfrac{H_y - H_{y1}}{T_L - T_{L1}} = \dfrac{L C_L}{G}}$

If L G known, can calculate T_{L2} in gas

maximum slope (max L flow allowed)

analogous to min L' in 2-phase

What else do we need? Calculate height Z that needs to have the H_{yi}, T_i at interface



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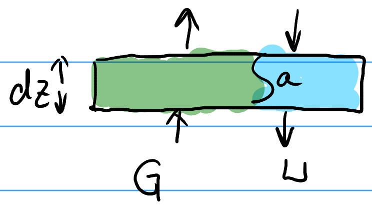
Design of cooling tower : interfacial profile

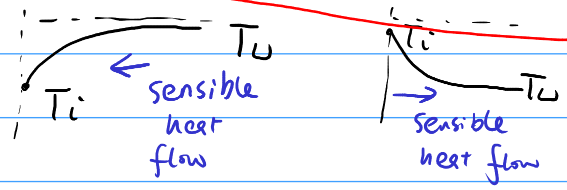
From last lecture, we need to have the slope to the E.L. from O.L to get (H_{yi}, T_i)

Recall how we solve 2-phase packed bed design (slab control volume)



O.D.E

$$\underbrace{G\,dH_y}_{\text{latent heat}} = \boxed{\underbrace{L c_L\, dT_L}_{\text{sensible heat}} = dz \cdot \underbrace{h_L a}_{\text{heat transfer coeff}} (T_L - T_i)}$$



What about LHS ?

For heat transfer to gas phase

$$G\,dH = \underbrace{\frac{q_s}{A}}_{\text{sensible}} + \underbrace{\frac{q_\lambda}{A}}_{\text{latent}} = h_G\, a\, dz\,(T_i - T_G) + M_B k_G a \lambda_0 (H_i - H_G)\, dz$$

$$= C_s \cdot P \cdot M_B k_G \cdot a\, dz\,(T_i - T_G) + M_B k_G a \lambda_0 (H_i - H_G)\, dz$$

$$\frac{h_G a}{M_B k_y a} \simeq C_s \qquad k_y = k_G \cdot P \Rightarrow h_G a = C_s \cdot M_B k_G \cdot P \cdot a$$

Rearrange L.H.S $\Rightarrow$

$$G\,dH = M_B k_G a P\, dz \left(\underbrace{C_s (T_i - T_0) + H_i \lambda_0}_{H_{yi}} - \underbrace{[C_s (T_G - T_0) + H_G \lambda_0]}_{H_y} \right)$$

$$= M_B k_G P a\, dz\, (H_{yi} - H_y)$$

Link to RHS

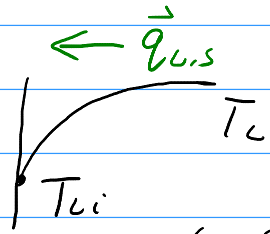
$$M_B k_G a P\, \cancel{dz}\, (H_{yi} - H_y) = h_L a (T_L - T_i)\, \cancel{dz}$$

R.H.S

unit: $L c_L\, dT_L$

$\frac{\cancel{kg}}{m^2 \cdot s}\cdot\frac{kJ}{\cancel{kg\,K}}\cdot d\cancel{K}$ $\Big\} \Rightarrow kJ/m^2\cdot s$ a heat flux

to link to a heat flux eq



$\vec{q} = h_L a\cdot(T_L - T_{Li})\,dz$ $\qquad (a\cdot S\cdot dz = A)$

effective contact area)

unit: $h_L a$ $\quad kJ/(s\,m^3 K) = kW/(m^3 K)$

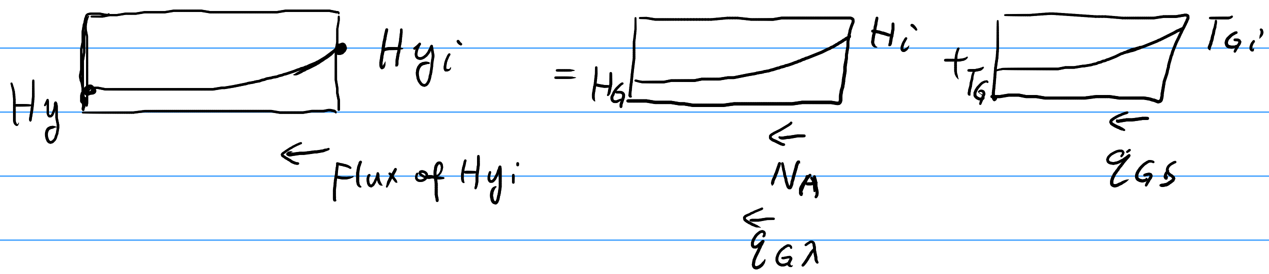
$h_L a\cdot\Delta T\cdot dz$ unit $\Rightarrow kW/(m^3\cdot K)\cdot K\cdot m \Rightarrow kW/m^2$

$= kJ/(m^2\cdot s)$

L.H.S

$G\,dH_y$ unit $\quad kg/(m^2\cdot s)\cdot kJ/kg = kJ/(m^2\cdot s)$

also a heat flux $= q_{G,S} + q_{G,\lambda}$



$q_{GS} = h_G a\,dz\,(T_i - T_G)$ (similar to Liquid)

$q_{G\lambda} = \lambda_0\cdot N_A\cdot M_A = \lambda_0\cdot k_y a\,dz\,(y_i - y_G)\cdot M_A$

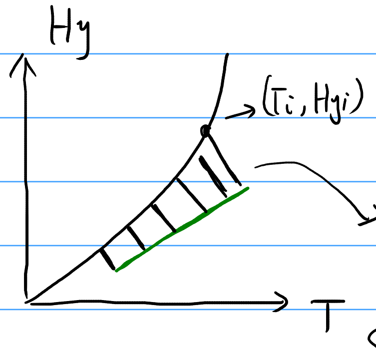
$y = \frac{M_B}{M_A}H$

$= \lambda_0\cdot \underbrace{k_y a}_{\Downarrow\ \text{the same as } k_G a P}\, dz\,(H_i - H_G)\cdot\frac{M_B}{\cancel{M_A}}\cdot\cancel{M_A}$

$= \underset{M_B}{\wedge}\lambda_0\, k_G a P\, dz\,(H_i - H_G)$

$$M_B k_G a P \cancel{dz} (H_{yi} - H_y) = h_L a (T_L - T_i) \cancel{dz}$$

$$\Downarrow$$



$$\frac{H_{yi} - H_y}{T_L - T_i} = \frac{h_L a}{M_B k_G a \cdot P}$$

Slope $= -\dfrac{h_L a}{M_B k_G a \cdot P}$

Since $\dfrac{h_L a}{k_G a}$ is usualy independent of the E.L. position (T, H), no iteration needed.

Solve for total tower height Z?

Use the relation

$$G \underbrace{dH}_{\text{Bulk, alone E.L.}} = M_B k_G \cdot a \cdot P dz \underbrace{(H_{yi} - H_y)}_{\text{dependent on local } H_y}$$

$$Z = \frac{G}{M_B k_G a P} \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y}$$

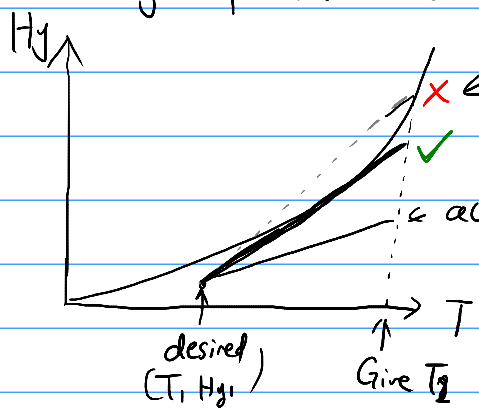
Can we use log-mean? No, as log mean statement needs H_{yi} & H_y lines to be both linear

Again, we can identify two components

$\dfrac{G}{M_B k_G a \cdot P} \Rightarrow H_G$ (height of transfer unit)

$\displaystyle\int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y} \Rightarrow N_G$ (number of transfer units)

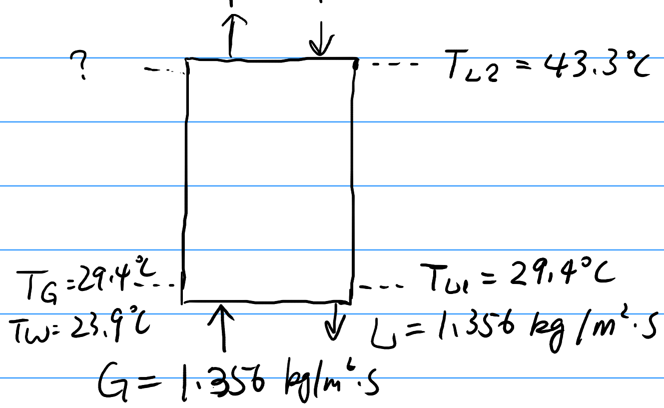
Design of tower caveat (min G flow)



Cannot use this line because it's within unphysical range.

actual flow $\frac{L c_L}{G_{min} * \text{factor}}$

Example from lecture 32



$k_G a = 1.207 \times 10^{-7} \frac{kg\,mol}{m^3 \cdot s \cdot Pa}$

$k_G a P = 0.0122 \frac{kg\,mol}{m^3 \cdot s}$

$\frac{h_L a}{k_G a P M_B} = 4.187 \times 10^4\ J/kg \cdot K$

$= 41.87\ kJ/kg \cdot K$

(to use our tool) $h_L a = 14.798\ kJ/(s\ m^3\ K)$

Be careful with units !

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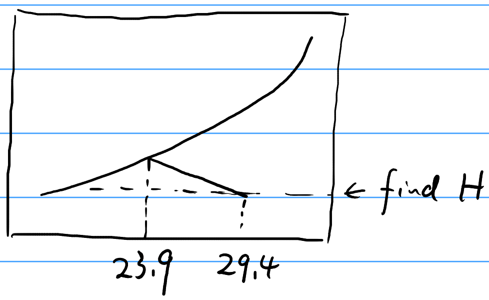
① Find humidity & enthalpy



Actually since the adiabatic sat. curve has same H_y

$T_W = 23.9°C \rightarrow H_s \rightarrow H_y$

From tool find H at 29.4°C ≈ 0.0165 (H_p = 62.9%)

$T_W = 23.91°C$ $H_y = 71.73\ kJ/kg\ air$

② Use cooling tower example

- Temp range in chart (°C)

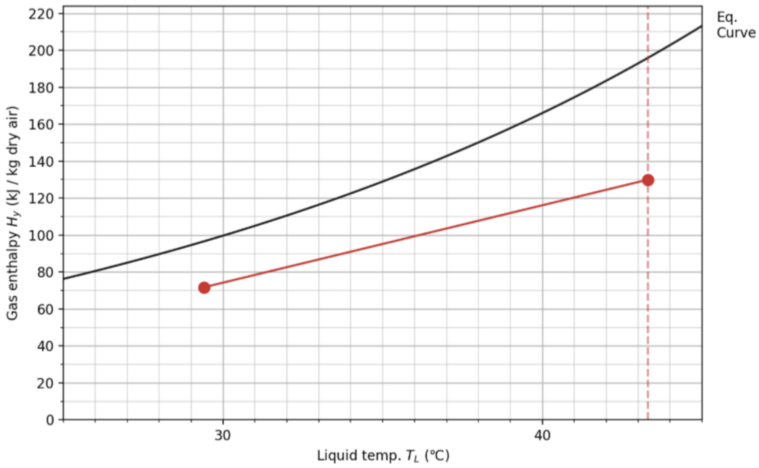
  25, 45
- T_{L1} (°C)

  29.4
- H_{y1} (kJ/kg):

  71.73
- T_{L2} (°C):

  43.3
- $h_L \cdot a$ (kJ/m^3 s K):

  14.8



For this system, seems 3 ~ 5 points are good enough for integration of Z

We get $Z = 7.03$ m from textbook (solved over 40 years ago) $Z = 6.98$ m

Solving T_G side?

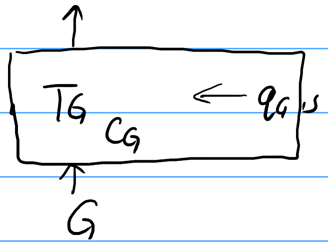
$$\overleftrightarrow{q_{G,s}} = h_G a\, dz\, (T_{Gi} - T_G)$$

$$\overrightarrow{q_{L,s}} \qquad \overrightarrow{q_{L,\lambda}} = M_B k_G a P\, dz\, (H_i - H_G)\lambda_0$$

$$\Downarrow$$

$$h_L a\, dz\, (T_L - T_{Li})$$

Solution: for gas-phase alone, change of T_G (bulk) is due to sensible heat (latent heat only change interfaces)



Sensible heat flux $\quad C_s \cdot G \cdot dT_G = h_G a\, dz\, (T_{Gi} - T_G)$

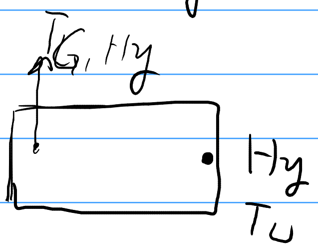
$$= h_G a\, dz\, (T_{Li} - T_G)$$

Compare $\quad C_L L\, dT_L = M_B k_G a P\, dz\, (H_{yi} - H_y)$

We get $\quad \dfrac{\cancel{C_s} G}{C_L L} \dfrac{dT_G}{dT_L} = \dfrac{h_G a}{M_B k_G a P} \dfrac{T_{Li} - T_G}{H_{yi} - H_y}$

$$\Downarrow$$

$$\cancel{C_s}$$



$$dT_G = \frac{L C_L}{G} \cdot \frac{T_{Li} - T_G}{H_{yi} - H_y} dT_L$$