CHE 318 Lecture 32

Cooling Tower Design (III)

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Learning outcomes

After this lecture, you will be able to:

- **Recall** the form of the cooling-tower height equation from coupled heat- and mass-transfer balances.
- **Describe** interfacial enthalpy driving forces using the enthalpy-temperature chart.
- **Apply** the design procedure to estimate tower height for a representative cooling-tower problem.
- **Analyze** how the model can be extended to predict the bulk-gas temperature profile.

Cheatsheet for cooling tower

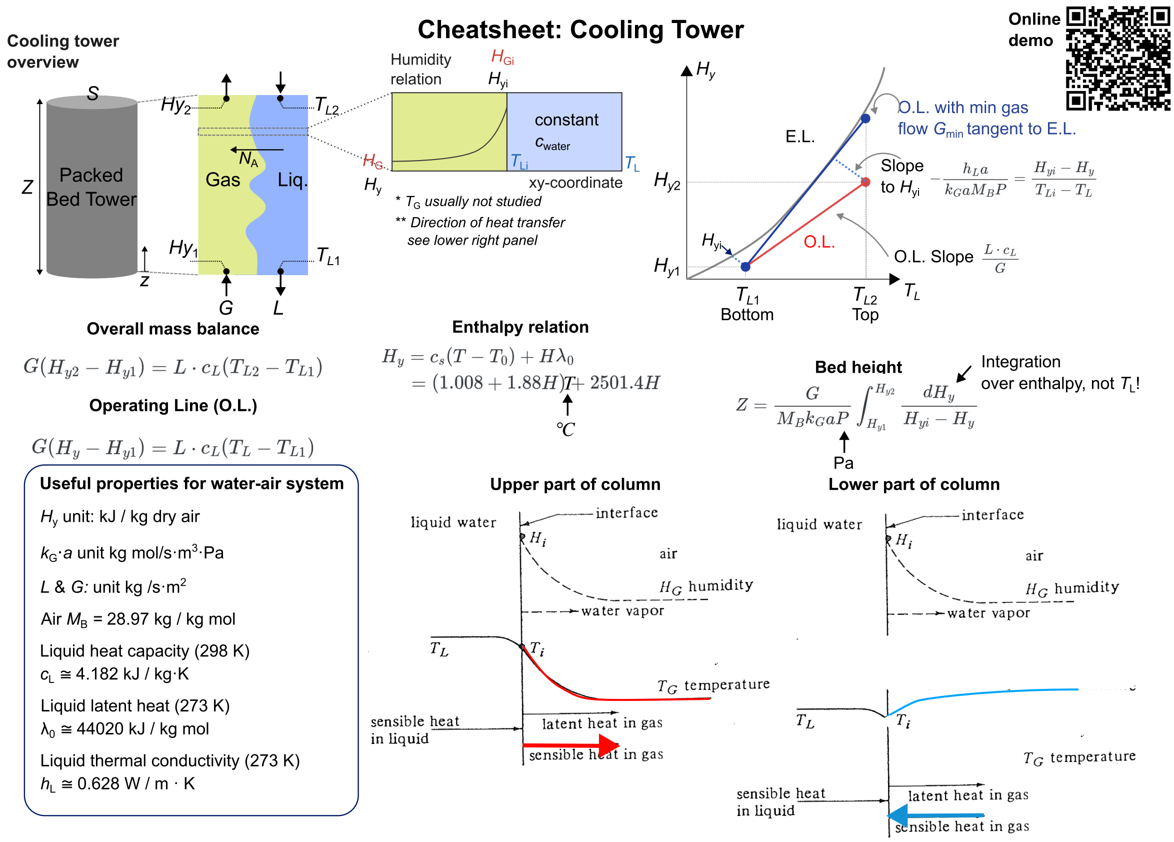

Figure 1: Charts distributed in class.

Recap: interfacial energy transfer

We wanted to solve the differential form

$$Lc_L dT_L = h_L a(T_L - T_{Li})dz \tag{1}$$

- L.H.S.: sensible heat in liquid $Lc_L dT_L = h_L a(T_L - T_{Li})dz$
- R.H.S.: sensible + latent heat in gas

Recap: solving the energy transfer in gas-phase

For the latent heat $q_{G,\lambda}$, it is solved by

$$\begin{aligned} q_{G,\lambda} &= M_A N_A \lambda_0 && (2)\\ &= M_A k_y a dz (y_i - y)\lambda_0 && (3)\\ &\approx M_A k_G a P dz \frac{M_B}{M_A}(H_i - H)\lambda_0 && (4)\\ &= M_B \lambda_0 k_G a P dz (H_i - H_G) && (5) \end{aligned}$$

- The derivation for $q_{G,\lambda}$ uses the fact $y \approx \frac{M_B}{M_A} H$
- Pressure-based coefficient $k_G a$ often used instead of $k_y a$

Energy transfer in liquid: final results

Adding the sensible & latent heat in liquid side gives

$$\begin{aligned} G dH_y &= q_{G,S} + q_{G,\lambda} && (6)\\ &= h_G a dz (T_i - T_G) + \lambda_0 a N_A M_A && (7)\\ &= h_G a dz (T_i - T_G) + M_B \lambda_0 k_G a P dz (H_i - H_G) && (8) \end{aligned}$$

As can be expected, both temperature and humidity driving forces should exhist.

Energy transfer in liquid: adiabatic process

Since the evaporation at interface is similar to the adiabatic process, the following relation (see Lecture 29) can be used:

$$\frac{h_G a}{M_B k_y a} = \frac{h_G a}{M_B k_G P} \approx c_s$$

which gives heat transfer in gas phase as

$$\begin{aligned} G dH_y &= c_s M_B k_G a P dz (T_i - T_G) + M_B \lambda_0 k_G a P dz (H_i - H_G) && (9)\\ &= M_B k_G a P dz \left[c_s (T_i - T_G) + \lambda_0 (H_i - H_G)\right] && (10)\\ &= M_B k_G a P dz (H_{yi} - H_y) && (11) \end{aligned}$$

Heat transfer at interfaces: implications

What are the implications for the follong equation?

$$GdH_y = M_B k_G aPdz(H_{yi} - H_y) \tag{12}$$

- The enthalpy in the gas phase H_y has an associated "transfer coefficient" $M_B k_G aP$!
- That justifies our choice of Enthalpy - Temperature chart.
  - **Enthalpy driving force** in gas phase
  - **Temperature driving force** in liquid phase

Interfacial flux equation for cooling tower

Combining the L.H.S with R.H.S we get

$$M_B k_G aPdz(H_{yi} - H_y) = h_L a(T_L - T_{Li})dz \tag{13}$$

$$\frac{H_{yi} - H_y}{T_{Li} - T_L} = -\frac{h_L a}{M_B k_G aP} \tag{14}$$

- The slope to find interfacial (H_{yi}, T_{Li}) is $-\frac{h_L a}{M_B k_G aP}$
- No longer need to do iterative slope searching, only 1 calculation!

Link to humidity chart adiabatic saturation curves

Recall in Lecture 29, the slope of adiabatic curves in psychrometric chart is given by

$$\frac{H_w - H}{T_w - T} = -\frac{h_G}{M_B k_y \lambda_w}$$

On the other hand the slope to find interfacial (H_{yi}, T_{Li}) in cooling tower is

$$\frac{H_{yi} - H_y}{T_{Li} - T_L} = -\frac{h_L a}{M_B k_G aP}$$

They have very similar forms, but be careful one is purely in gas phase and the other describes the 2-phase equilibrium.

Solving the tower height

If only use the R.H.S result

$$G dH_y = M_B k_G a P dz (H_{yi} - H_y) \tag{15}$$

we can obtain the total tower height by integration

$$Z = \int_0^Z dz \tag{16}$$

$$= \int_{H_{y1}}^{H_{y2}} \frac{G}{M_B k_G a P} \frac{dH_y}{H_{yi} - H_y} \tag{17}$$

> ⚠ Warning
>
> The integral is carried out over dH_y, not T_L!

Steps to solve the cooling tower

Similar to absorption tower, cooling tower design typically involves the following steps

1. Plot the saturated air enthalpy H_{yi} vs liquid temperature T_L (usually a given chart)
2. Knowing the entering air conditions T_{G1} and H_1 (humidity), calculate the enthlpy H_{y1} (not saturated).
3. The tower bottom (T_{L1}, H_{y1}) on the operating line is determined from T_{L1} requirement
4. Find the minimal gas flow rate G_{min} and practical operating G, determine tower top operating line point (T_{L2}, H_{y2})
5. Know the $h_L a$ and $k_G a P$ (or $k_y a$) values, pick several points along the operating line, use slope $-\frac{h_L a}{M_B k_G a P}$ to find H_{yi} and calculate $1/(H_{yi} - H_y)$
6. Numerically integrate $\int_{H_{y1}}^{H_{y2}} 1/(H_{yi} - H_y) dH_y$ and $\frac{G}{M_B k_G a P}$ to find Z

Cooling tower design problem example

EXAMPLE 10.5-1. Design of Water-Cooling Tower Using Film Coefficients A packed countercurrent water-cooling tower using a gas flow rate of $G = 1.356$ kg dry air/s · m^2 and a water flow rate of $L = 1.356$ kg water/s · m^2 is to cool the water from $T_{L2} = 43.3$°C (110°F) to $T_{L1} = 29.4$°C (85°F). The entering air at 29.4°C has a wet bulb temperature of 23.9°C. The mass-transfer coefficient $k_G a$ is estimated as 1.207×10^{-7} kg mol/s · m^3 · Pa and $h_L a/k_G a M_B P$ as 4.187×10^4 J/kg · K (10.0 btu/lb$_m$ · °F). Calculate the height of packed tower z. The tower operates at a pressure of 1.013×10^5 Pa.

> **Note**
>
> A similar example is given in Assignment 8. Be careful about the units!

Finding the humidity of inlet air

Interfacial demo

Textbook solution (1)

Solution: Following the steps outlined, the enthalpies from the saturated air–water vapor mixtures from Table 10.5-1 are plotted in Fig. 10.5-4. The inlet air at $T_{G1} = 29.4°C$ has a wet bulb temperature of 23.9°C. The humidity from the humidity chart is $H_1 = 0.0165$ kg H_2O/kg dry air. Substituting into Eq. (9.3-8), noting that (29.4 − 0)°C = (29.4 − 0) K,

$$H_{y1} = (1.005 + 1.88 \times 0.0165)10^3(29.4 - 0) + 2.501 \times 10^6(0.0165)$$
$$= 71.7 \times 10^3 \text{ J/kg}$$

The point $H_{y1} = 71.7 \times 10^3$ and $T_{L1} = 29.4°C$ is plotted. Then substituting into Eq. (10.5-2) and solving,

$$1.356(H_{y2} - 71.7 \times 10^3) = 1.356(4.187 \times 10^3)(43.3 - 29.4)$$

$H_{y2} = 129.9 \times 10^3$ J/kg dry air (55.8 btu/lb_m). The point $H_{y2} = 129.9 \times 10^3$ and $T_{L2} = 43.3°C$ is also plotted, giving the operating line. Lines with slope $-h_L a / k_G a M_B P = -41.87 \times 10^3$ J/kg·K are plotted giving H_{yi} and H_y values, which are tabulated in Table 10.5-2 along with derived values as shown. Values of $1/(H_{yi} - H_y)$ are plotted versus H_y and the area under the curve from $H_{y1} = 71.7 \times 10^3$ to $H_{y2} = 129.9 \times 10^3$ is

$$\int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y} = 1.82$$

Textbook solution (2)

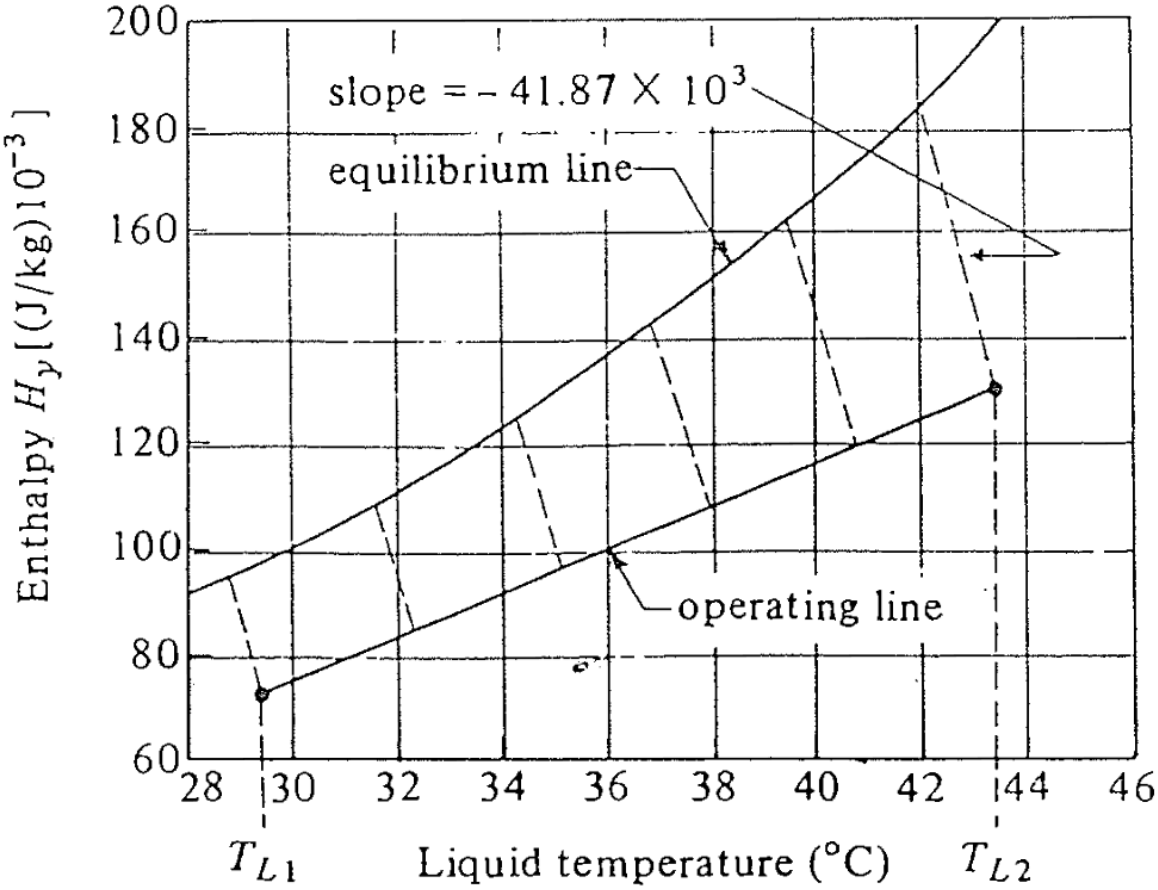

FIGURE 10.5-4. *Graphical solution of Example 10.5-1.*

Textbook solution (3)

We get $Z = 7.03$ m, pretty close!

TABLE 10.5-2. *Enthalpy Values for Solution to Example 10.5-1 (enthalpy in J/kg dry air)*

H_{yi}	H_y	$H_{yi} - H_y$	$1/(H_{yi} - H_y)$
94.4×10^3	71.7×10^3	22.7×10^3	4.41×10^{-5}
108.4×10^3	83.5×10^3	24.9×10^3	4.02×10^{-5}
124.4×10^3	94.9×10^3	29.5×10^3	3.39×10^{-5}
141.8×10^3	106.5×10^3	35.3×10^3	2.83×10^{-5}
162.1×10^3	118.4×10^3	43.7×10^3	2.29×10^{-5}
184.7×10^3	129.9×10^3	54.8×10^3	1.82×10^{-5}

Substituting into Eq. (10.5-13),

$$z = \frac{G}{M_B k_G a P} \int \frac{dH_y}{H_{yi} - H_y} = \frac{1.356}{29(1.207 \times 10^{-7})(1.013 \times 10^5)} (1.82)$$

$$= 6.98 \text{ m } (22.9 \text{ ft})$$

Final question: what if we wanted to know the T_G as well?

Advanced discussion. May not appear in final exam.

For the change of bulk-gas temperature T_G, the following energy flux equation can be given for the sensible heat in gas $q_{G,S}$:

$$c_s G dT_G = h_G a dz(T_{Gi} - T_G) \tag{18}$$

$$= h_G a dz(T_{Li} - T_G) \tag{19}$$

For comparison we also have the change of bulk liquid temperature expressed as

$$c_L L dT_L = M_B k_G a P dz(H_{yi} - H_y) \tag{20}$$

Our goal is to find a differential equation so that T_G can be integrated from T_{G1} (cool air intake)

Solving the ODE for T_G

Combine the energy balance equations for T_G and T_L gives

$$\frac{c_L L}{c_s G}\frac{dT_L}{dT_G} = \frac{M_B k_G a P}{h_G a}\frac{H_{yi} - H_y}{T_{Li} - T_G} \tag{21}$$

$$dT_G = \frac{c_L L}{G}\frac{T_{Li} - T_G}{H_{yi} - H_y} dT_L \tag{22}$$

- Again, we cancel out $c_S = \frac{h_G a}{M_B k_G a P}$. This is an
- integrable ODE, given that at any (T_L, H_y), we can calculate H_{yi} without needing T_G

Integrating T_G: graphical explanation

Summary

- Cooling-tower height can be obtained by integrating the interfacial enthalpy driving force across the column.
- The enthalpy-temperature chart provides the graphical information needed to evaluate interfacial states.
- The same framework can be extended to estimate the bulk-gas temperature profile when needed.