

CHE 318 Lecture 33

Final Exam Review Lecture (I)

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Learning outcome

This is a review lecture for preparing for the final exam, focusing on the concept sheets from pre-midterm.

Cheatsheet for pre-midterm mass transfer

Cheatsheet: Pre-midterm Mass Transfer

1D Equation. Other forms see below

Flux Equation (Solving N_A)

$$N_A = -c_T D_{AB} \frac{dx_A}{dz} + x_A (N_A + N_B)$$

Diffusive J_{Az}^* Convective $c_A v_m$

Solutions Gas, Steady State (Gen=Acc=0)

General solution

$$N_A = \frac{c_T D_{AB}}{(z_2 - z_1)} \frac{N_A}{N_A + N_B} \ln \left[\frac{N_A - x_{A2}(N_A + N_B)}{N_A - x_{A1}(N_A + N_B)} \right]$$

Suitable: boundary of heterogeneous reaction

Special case 1: EMCD

$$N_A = \frac{D_{AB}}{(z_2 - z_1)} (c_{A1} - c_{A2})$$

Special case 2: Through stagnant B

$$N_A = \frac{c_T D_{AB}}{(z_2 - z_1)} \ln \left(\frac{1 - x_{A2}}{1 - x_{A1}} \right)$$

Use log-mean format

$$N_A = \frac{D_{AB}}{RT(z_2 - z_1)} \frac{p_T}{p_{Bm}} (p_{A1} - p_{A2})$$

Special case 3: reactive coupling

Use general solution, need N_A/N_B ratio & sign

$\begin{matrix} N_A \leftarrow \\ N_B \rightarrow \end{matrix}$

$\begin{matrix} N_A \rightarrow \\ N_B \rightarrow \end{matrix}$

Stoichiometry Magnitude ratio
 $mA \rightarrow nB$ $N_A/m = N_B/n$

sign: determined from boundary condition

Mass Balance Equation

[In] - [Out] + [Gen] = [Acc]

$$N_A(z)A(z) - N_A(z+\Delta z)A(z+\Delta z) + r_A A_s(\Delta z) = dc_A/dt$$

Control Volume

Generation term / orthogonal M.T.

Coefficient - driving force form
 Example: (details see review after mid-term)

$$r_A = k(c_{A,s} - c_A)$$

Coeff Driving force
 Unit m/s Concentration diff

Unsteady state $dc_A/dt \neq 0$

1D uniform cross sectional area (remove Δz)

$$\frac{\partial c_A}{\partial t} = -\frac{\partial N_A(z)}{\partial z} + r_A$$

1D

$$\nabla^2 c = \frac{\partial^2 c}{\partial z^2}$$

3D general term

$$\frac{\partial c_A}{\partial t} = r_A - \frac{\partial N_{Ax}}{\partial x} - \frac{\partial N_{Ay}}{\partial y} - \frac{\partial N_{Az}}{\partial z}$$

$$= r_A - \nabla \cdot \vec{N}_A$$

Most used form: incompressible liquid

$$\frac{\partial c_A}{\partial t} = D_{AB} \nabla^2 c_A - \vec{v}_m \cdot \nabla c_A + r_A$$

likely diffusion can be ignored

Varying cross section problem

N_A can vary! Steady state use $\bar{N}_A = N_A(z)A(z)$

Gen = 0. Spherical case (stagnant B)

$$\frac{\bar{N}_A}{4\pi} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{D_{AB} p_T}{RT} \ln \left(\frac{p_T - p_{A2}}{p_T - p_{A1}} \right)$$

Gen = 0. Spherical case (far field stagnant B)

$$N_{A1} = \frac{D_{AB} p_T}{RT p_{Bm}} (p_{A1} - p_{A2})$$

Diffusivity D_{AB}

Gas: Kinetic theory Liq: Stokes-Einstein (Mw > 1000)

$$D_{AB} = \frac{1}{3} \bar{u} \lambda_{A,B}$$

$$D_{AB} = \frac{9.96 \times 10^{-16} T}{\eta V_A^{0.333}}$$

Gas: Chapman-Enskog Liq: Wilke-Chang (278 K ~ 313 K)

$$D_{AB} = \frac{1.8583 \times 10^{-7} T^{3/2}}{p_T \sigma_{AB}^2 \Omega_{D,AB}} \left(\frac{1}{m_A} + \frac{1}{m_B} \right)^{1/2}$$

$$D_{AB} = \frac{1.173 \times 10^{-16} (\phi M_B)^{1/2} T}{\eta_B V_A^{0.6}}$$

Gas: Fuller method Inhom. Solid: effective

$$D_{AB} = \frac{1.0 \times 10^{-7} T^{1.75}}{p_T (\sum \nu_A)^{1/3} + (\sum \nu_B)^{1/3}} \left(\frac{1}{m_A} + \frac{1}{m_B} \right)^{1/2}$$

$$D_{AB,eff} = \frac{\epsilon}{\tau} D_{AB}$$

Liquid / solid common case (like stagnant B)

$$N_A = \frac{D_{AB} c_{A,av}}{(z_2 - z_1)} \frac{(x_{A1} - x_{A2})}{x_{B,m}} \quad x_{B,m} = \frac{x_{B2} - x_{B1}}{\ln(x_{B2}/x_{B1})}$$

Solid case $x_{Bm} \sim 1$
 Liq. dilute case $x_{Bm} \sim (x_{B1} + x_{B2})/2$

Figure 1: Charts distributed in class. PDF Version

Cheatsheet for convective mass transfer & coefficients

Cheatsheet: Convective Mass Transfer

Q1: Interfacial equilibrium

$$K = \frac{[\text{Conc. at gas side}]}{[\text{Conc. at liquid side}]} = \frac{c_i}{c_{Li}}$$

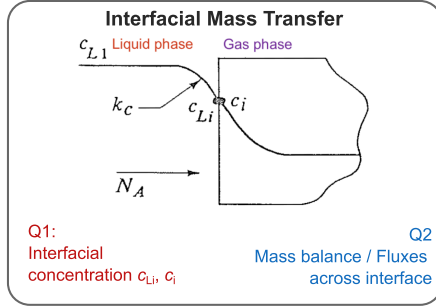
Example:

1. Henry's Law $p_A = H x_A$

2. Solubility (in liquid / solid)

$$c_A = S p_A / 22.414 \quad (S \text{ unit: } m_3 \text{ at S.T.P})$$

3. Equilibrium diagram (see later topics)



Q2: Flux equation in convective regime

$$N_A = k_c (c_{L,b} - c_{L,i})$$

Mass balance = Flux matching
- applicable to 2-phase M.T. / cooling

$$N_A|_{\text{liq}} = N_A|_{\text{gas}} \\ k_{c,\text{liq}} (c_{L,b} - c_{L,i}) = k_{c,\text{gas}} (c_{g,i} - c_{g,b})$$

Gas phase

$$k'_c c_T = k'_c \frac{p_T}{RT} = k_c \frac{p_{Bm}}{RT} \\ = k'_G p_T = k_G p_{Bm} \\ = k'_y = k_y y_{Bm} \\ = k_c y_{Bm} c_T = k_G y_{Bm} p_T$$

- p : total pressure
- p_{Bm} : log-mean partial pressure of inert B
- y_{Bm} : log-mean mole fraction of B
- $c_T = p_T / (RT)$

Liquid phase

$$k'_c c = k'_L c = k_L x_{Bm} c \\ = k'_L \frac{\rho}{M} = k'_x = k_x x_{Bm}$$

- ρ : liquid density
- M : molecular weight
- x_{Bm} : log-mean mole fraction of solvent B

Naming Convention and Units

- Superscript: EMCD $k'_{\text{driving force}}$; Convective / stagnant $k_{\text{driving force}}$

Phase / Driving force	Concentration c_A	Partial pressure p_A	Mole fraction (gas y_A , liquid x_A)
Gas phase	k_c, k'_c	k_G, k'_G	k_y, k'_y
Liquid phase	k_c, k'_c	-	k_x, k'_x
Liquid (alt. form)	k_L, k'_L	-	-
Unit of k	$m \cdot s^{-1}$	$\frac{\text{kg mol}}{\text{s} \cdot \text{m}^2 \cdot \text{Pa}}$	$\frac{\text{kg mol}}{\text{s} \cdot \text{m}^2 \cdot \text{mol frac}}$

EMCD / Dilute

Gases: $N_A = k'_c (c_{A1} - c_{A2}) = k'_G (p_{A1} - p_{A2}) = k'_y (y_{A1} - y_{A2})$
Liquids: $N_A = k'_c (c_{A1} - c_{A2}) = k'_L (c_{A1} - c_{A2}) = k'_x (x_{A1} - x_{A2})$

Stagnant film

Gases: $N_A = k_c (c_{A1} - c_{A2}) = k_G (p_{A1} - p_{A2}) = k_y (y_{A1} - y_{A2})$
Liquids: $N_A = k_c (c_{A1} - c_{A2}) = k_L (c_{A1} - c_{A2}) = k_x (x_{A1} - x_{A2})$

Theory 1: Thin film theory

$$N_A = \frac{D_{AB}}{\delta_f} (c_{A,i} - c_{A,b})$$

Theory 2: Penetration theory

$$N_A = \sqrt{\frac{A D_{AB}}{\pi t_L}} (c_{A,s} - c_{A,b})$$

Theory 3: Boundary layer theory

$$N_A = 0.664 D_{AB} / L N_{Sc}^{1/3} N_{Re}^{1/2} (c_{A,s} - c_{A,b})$$

Practical: use cheatsheet for scaling

Figure 2: Charts distributed in class. PDF Version